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An Application of Information Theory to the Analysis of Contingency Tables, With a Table of $2n \ln n$, $n=1(1)10{,}000$

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In this paper we present a number of useful tests for contingency tables in conjunction with a useful table to assist in the necessary computations. A consistent and simple approach based on the notions of information theory is used in developing the various test procedures and the results are analyzed in the form of analysis-of-information tables. Beginning with tests of hypotheses for a one-way table, tests of hypotheses of specified probabilities, independence, conditional independence, homogeneity of classifications, symmetry, and interaction are developed or indicated for contingency tables of two, three, four, and higher-order classifications. Extension of these procedures to certain tests for Markov chains is indicated. Worked examples are given throughout the paper. A table of $2n \ln n$ for $n=1(1)10{,}000$ is appended for use in computation.

1. Introduction

The χ^2 test of goodness of fit, first introduced by Karl Pearson (1900), has been utilized for the analysis of sequences of observed categorical data for certain properties. Modifications and extensions of this work followed; for expository reviews of the developments see Cochran (1952), Lewis (1962).

In this paper we present a table of $2n \ln n$ and, in a form suitable for use of the table, a variety of statistical tests of various hypotheses on one or several observed sequences. We propose to show the conceptual simplicity of an approach, using the notions of information theory, to the statistical analyses of contingency tables, and particularly the computational convenience of the resulting procedures. It is our hope that this exposition will contribute to a wider interest in, and application and use of, these procedures.

A contingency table is essentially a sample from a multivalued population with the various probabilities and partitions of the categories subject to restrictions in addition to those of the multinomial distribution. The analyses of contingency tables may therefore be closely related to those of multinomial samples. The procedures proposed depend on the use of a minimum discrimination information statistic (m.d.i.s.) and its asymptotic distribution properties. A complete theoretical background and applications of this statistic to a variety of statistical problems may be found in Kullback (1959). For multinomial samples in particular, we present the following brief summary of the underlying philosophy.

Consider the n-total multinomial distribution on a population of c categories or classes

$$p(x)=p(x_1,x_2, \ldots, x_c)=\frac{n!}{x_1! \ldots x_c!} p_1^{x_1} \ldots p_c^{x_c}, \quad (1.1)$$

where

$$p_i>0,\ i=1,2, \ldots, c, \qquad p_1+p_2+ \ldots +p_c=1,$$
$$x_1+x_2+ \ldots +x_c=n.$$

Suppose that $p^*(x)$ is any other distribution on the population of c categories such that every possible observation from $p^*(x)$ is also a possible observation from $p(x)$. Further, if we define the discrimination information gained from an observation x to be the logarithm of the likelihood ratio $\ln p^*(x)/p(x)$, and the mean discrimination information $I(p^*:p)$ to be the expected value of this quantity with respect to $p^*(x)$, then we can state the following result.

The least informative distribution on the population of c categories, with given expected values for discrimination against the multinomial distribution $p(x)$ in (1.1), namely, the distribution $p^*(x)$ such that $E^*(x_i)=\theta_i$ and

$$I(p^*:p)=\sum_{x_1+ \ldots +x_c=n} p^*(x) \ln \frac{p^*(x)}{p(x)}$$

is a minimum, is the distribution

$$p^*(x)=e^{t_1x_1+ \ldots +t_cx_c}p(x)/(p_1e^{t_1}+ \ldots +p_ce^{t_c})^n \quad (1.2)$$
$$=\frac{n!}{x_1! \ldots x_c!} (p_1^*)^{x_1} \ldots (p_c^*)^{x_c},$$

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where $p_i^*=p_ie^{t_i}/(p_1e^{t_1}+\ldots+p_ce^{t_c})$, $i=1,2,\ldots,c$, the t's are real parameters, and $\theta_i=(\partial/\partial t_i)\ln(p_1e^{t_1}+\ldots+p_ce^{t_c})^n$. The minimum value, $\min I(p^*:p)$ is then

$$\min I(p^*:p)=\theta_1\ln\frac{\theta_1}{np_1}+\ldots+\theta_c\ln\frac{\theta_c}{np_c}. \quad (1.3)$$

The minimum discrimination information statistic is

$$2\hat{I}=2\left(\hat{\theta}_1\ln\frac{\hat{\theta}_1}{np_1}+\ldots+\hat{\theta}_c\ln\frac{\hat{\theta}_c}{np_c}\right), \quad (1.4)$$

where $\hat{\theta}_i$ is the minimum variance unbiased estimate of θ_i in the sample.

Heuristically the m.d.i.s. can be considered as a measure of the "divergence" of the alternative hypothesis, as evidenced by the sample values, from the null hypothesis as evidenced by $p(x)$, and in the sense of minimum information for discrimination.

It can be shown that the m.d.i.s.

(a) is distributed asymptotically as χ^2 under the null hypothesis, and as noncentral χ^2 under the alternative hypothesis, with appropriate degrees of freedom and noncentrality parameter;

(b) has additive properties;

(c) has the convexity property.

In particular, for nonnegative real numbers a_i and b_i, the convexity property yields

$$a_1\ln\frac{a_1}{b_1}+a_2\ln\frac{a_2}{b_2}+\ldots+a_n\ln\frac{a_n}{b_n} \quad (1.5)$$

$$\geqq(a_1+a_2+\ldots+a_n)\ln\frac{a_1+a_2+\ldots+a_n}{b_1+b_2+\ldots+b_n},$$

with equality if and only if $a_1/b_1=\ldots=a_n/b_n$. The convexity property is very useful in finding the minimum value of information statistics under certain restrictions and groupings, thus leading to χ^2-distributions (central and noncentral) with appropriate reductions in degrees of freedom.

For the categorical type of data we shall be considering, the m.d.i.s. in its simplest form is (cf. Kullback (1959, p. 113))

$$2\hat{I}=2\sum_{i=1}^{c}f_i\ln\frac{f_i}{np_i}, \quad (1.6)$$

where p_i is the probability of an observation from the ith category under the null hypothesis, $p_1+p_2+\ldots+p_c=1$, f_i is the observed frequency of occurrence of the ith category, $f_1+f_2+\ldots+f_c=n$, and ln is the natural logarithm. We define $0\ln 0=0$.

If we write O for observed frequencies and E for expected frequencies, then, with $\sum_i O_i=\sum_i E_i$,

$$2\hat{I}=2\sum_i O_i\ln(O_i/E_i)\approx\sum_i(O_i-E_i)^2/E_i, \quad (1.7)$$

by use of the approximation $\ln\dfrac{O_i}{E_i}\approx\dfrac{1}{2}\dfrac{O_i^2-E_i^2}{O_iE_i}$ with $O_i>0$ and $E_i>0$ (cf. Kullback (1959, p. 114)). The last expression in (1.7) is the familiar χ^2 statistic.

The m.d.i.s. in the examples of this paper turns out to be equal to minus twice the natural logarithm of a likelihood ratio. A comparison of χ^2 and m.d.i.s. or likelihood ratio test is not the purpose of this paper. It is interesting to recall that Wilks (1935) remarked that there was no theoretical reason why χ^2 should be preferred to $-2\ln\lambda$ and that $-2\ln\lambda$ can be computed with fewer operations than χ^2. Cochran (1952) stated that, "In view of the equivalence of the two criteria in large samples, there seems no advantage, except one of taste or convenience, in one test over the other. For small samples, the suggestion has been made from time to time that the likelihood ratio is to be preferred . . . . However, not enough data about relative power has accumulated to permit a verdict on this issue." Good (1957) remarks that the m.d.i.s. or $-2\ln\lambda$ more closely puts the possible samples in order of their likelihoods under the null hypothesis, as compared with χ^2 for given $n,c,p_1,\ldots,p_c$; the calculation of m.d.i.s. or $-2\ln\lambda$ can be done by additions, subtractions, and table look-ups only, when tables of $2n\ln n$ are available, but the calculation is less "well-conditioned" than for χ^2, in the sense that more significant figures must be held; χ^2 is a simpler mathematical function of the observations and it should be easier to approximate closely to its distribution, given the null hypothesis. Anderson and Goodman (1957) discussed the asymptotic equivalence of the two statistics under the null hypothesis for Markov chains, and have also discussed possible criteria for the selection of one over the other.

The utility of the m.d.i.s., however, lies in its additivity, convexity, and computational properties. As we shall see, the m.d.i.s. can be analyzed into several additive components, similar to the analysis of variance, for a hypothesis that is equivalent to the combination of several hypotheses of interest. Each component of the m.d.i.s. is itself an m.d.i.s. and asymptotically distributed as χ^2 with appropriate degrees of freedom.

Examples of the application of some of the statistical tests are given throughout the paper. Applications of the m.d.i.s. to the analysis of intraclass contingency tables are given by Ishii (1960).

2. One-Way Tables

Suppose that each observation from a population can be classified into exactly one of $c(\geq 2)$ categories.

Consider a random sample of n independent observations from such a c-valued population, with the observations denoted by $x_1, x_2, \ldots, x_n$, where x_i takes on one of c possible values. A common statistical procedure is to test a null hypothesis of equidistribution, that is, $p_i=1/c$ $(i=1,2,\ldots,c)$, where p_i is the probability of an observation in the ith category.

Let f_i represent the observed frequency of the ith category in the sample, so that $f_1+f_2+\ldots+f_c=n$. The appropriate m.d.i.s. is that given in (1.6) with $p_i=1/c$, or

$$2\hat{I}=2\sum_{i=1}^{c} f_i \ln \frac{cf_i}{n}. \tag{2.1}$$

$2\hat{I}$ in (2.1) is asymptotically distributed as χ^2 with $c-1$ degrees of freedom under the null hypothesis. If the probability of exceeding the computed value of $2\hat{I}$ is small according to the χ^2-distribution for $c-1$ degrees of freedom, the null hypothesis is rejected.

Note that (2.1) may be written as

$$2\hat{I}=2\sum_{i=1}^{c} f_i \ln f_i - 2n \ln n + 2n \ln c \tag{2.2}$$

for computational convenience. Use may be made of tables of $n \ln n$ and $\ln n$, for example in Kullback (1959) (up to $n=1000$) and Masuyama (1960), or of the table of $2n \ln n$ in Woolf (1957) (up to $n=2009$), or at the end of this paper (up to $n=10{,}000$). Essentially table look-up, addition, and subtraction are all that is necessary.

We illustrate (2.2) with an example from Hald (1952). In reading a scale, where the last figure is estimated, it is often seen that the observer prefers certain figures to others. Table 2.1 gives the observed distribution of 200 readings of an instrument by a certain observer according to magnitude of the last figure.

Table 2.1

Last figure	0	1	2	3	4	5	6	7	8	9	Total
Obs. number	35	16	15	17	17	19	11	16	30	24	200

The null hypothesis to be tested is that of equidistribution, or that the observer did not have any preference for certain figures. The m.d.i.s. is

$$2\hat{I}=2(35 \ln 35)+2(16 \ln 16)+ \ . \ . \ . \ +2(24 \ln 24) -2(200 \ln 200)+2(200 \ln 10)=23.19.$$

Since there are 9 degrees of freedom, and the probability of obtaining a χ^2 as large as 23.19 under the null hypothesis is less than 0.01, we may conclude that the observer is probably biased towards certain figures. Hald computed a χ^2 value of 24.90.

A more general problem is to test the null hypothesis that an observed sequence was drawn randomly from a population whose theoretical probabilities are the specified numbers $p_1, p_2, \ldots, p_c$, where $\sum_{i=1}^{c} p_i=1$. The previous case of equidistribution is a special case of the present problem, with $p_i=1/c$, for $i=1, 2, \ldots, c$.

The appropriate m.d.i.s. is

$$2\hat{I}=2\sum_{i=1}^{c} f_i \ln \frac{f_i/n}{p_i}=\sum_{i=1}^{c} 2f_i \ln f_i - 2n \ln n - \sum_{i=1}^{c} 2f_i \ln p_i, \tag{2.3}$$

which is asymptotically distributed as χ^2 with $c-1$ degrees of freedom. A table of $-\ln p$ is given by Bartlett (1952) for $p=0(0.01)1.00$ to four decimal places. If Bartlett's table is not available, we may modify formula (2.3) above so that a table of $\ln n$ may be used by setting $p_i=u_i/a$, where numerator and denominator are integers and the denominator is common to all the p_i (for decimal fractions a would be some power of 10).

The following data represent the observed numbers of heads in a series of 74 independent tosses of five coins.

3232212433323242222334332322023433025334421431
2243223124332334434543334331

Assuming independence, we wish to test the null hypothesis of a binomial distribution with parameter 1/2, that is, if $p_i=\text{Prob}\{\text{number of heads}=i\}$, $i=0, 1, 2, 3, 4, 5$, then $p_0=p_5=1/32$, $p_1=p_4=5/32$, $p_2=p_3=10/32$, against an alternative hypothesis of any distribution into six categories.

Table 2.2 shows the observed frequencies and theoretical probabilities (given the null hypothesis):

Table 2.2

Number of heads	Observed frequency	Theoretical probability
0	2	1/32
1	5	5/32
2	22	10/32
3	29	10/32
4	14	5/32
5	2	1/32
Total	74	

For the data in table 2.2 the m.d.i.s. yields the value

$$2\hat{I}=2(2 \ln 2)+ \ . \ . \ . \ +2(2 \ln 2)+2(74 \ln 32) -2(2 \ln 1)- \ . \ . \ . \ -2(74 \ln 74)=6.748.$$

Since the value 6.748 as a χ^2 with 5 degrees of freedom will be exceeded with a probability of about 0.25, we may accept the null hypothesis of a binomial distribution with parameter 1/2.

The test for the homogeneity of one-way tables is discussed in the next section.

3. Two-Way Tables

Let us suppose now that each observation may be categorized by two criteria of classification. In a sample of n independent observations, each observation then consists of two parts, as follows:

$$(A_1, B_1), (A_2, B_2), \ . \ . \ . \ , (A_n, B_n).$$

Now assume that the first criterion of classification has r different categories and that the second criterion has c different categories. We form the $r \times c$ contingency table 3.1, where the entries in the body of the table are frequencies (integers).

TABLE 3.1

		Second criterion of classification (B)						
		1	2	--	j	--	c	Total
	1	f_{11}	f_{12}	--	f_{1j}	--	f_{1c}	$f_{1\cdot}$
	2	f_{21}	f_{22}	--	f_{2j}	--	f_{2c}	$f_{2\cdot}$
First	--	--	--	--	--	--	--	--
criterion	i	f_{i1}	f_{i2}	--	f_{ij}	--	f_{ic}	$f_{i\cdot}$
of classification	--	--	--	--	--	--	--	--
(A).	r	f_{r1}	f_{r2}	--	f_{rj}	--	f_{rc}	$f_{r\cdot}$
Total		$f_{\cdot 1}$	$f_{\cdot 2}$	--	$f_{\cdot j}$	--	$f_{\cdot c}$	$f_{\cdot\cdot}=n$

In order to test the null hypothesis that the two criteria of classification are independent of each other in the probability sense, against the alternative hypothesis that the criteria are not independent, we use the m.d.i.s.

$$2\hat{I}=\sum_{i=1}^{r}\sum_{j=1}^{c}2f_{ij}\ln f_{ij}+2n\ln n-\sum_{i=1}^{r}2f_{i\cdot}\ln f_{i\cdot}$$

$$-\sum_{j=1}^{c}2f_{\cdot j}\ln f_{\cdot j}, \tag{3.1}$$

which is distributed asymptotically as χ^2 with $(r-1)(c-1)$ degrees of freedom under the null hypothesis of independence. In addition to testing the two criteria of classification for independence, we may test the marginal row and column totals against null hypotheses with specified probabilities $p_{i\cdot}$ and $p_{\cdot j}$, respectively, $i=1, 2, \ldots, r$, $j=1, 2, \ldots, c$. The complete analysis is given in table 3.2.

We may arrange the analysis of the test statistics in a table similar in concept to an analysis-of-variance table, and we term the table an analysis-of-information table. The m.d.i.s. for the rc cells in the contingency table, for which f_{ij} and p_{ij} are respectively the observed frequency and theoretical probability for the cell in the ith row and jth column, is

$$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{np_{ij}}, \tag{3.2}$$

which is asymptotically distributed as χ^2 with $rc-1$ degrees of freedom under the null hypothesis that the cell probabilities are p_{ij}. This is a simple application of the theory of section 2, for we now have an rc-valued population. If the row (marginal) probabilities $p_{i\cdot}$ are specified, we use the m.d.i.s.

$$2\sum_{i=1}^{r}f_{i\cdot}\ln\frac{f_{i\cdot}}{np_{i\cdot}}, \tag{3.3}$$

which is asymptotically distributed as χ^2 with $r-1$ degrees of freedom. If the conditional probabilities $p_{ij}/p_{i\cdot}$ are specified, we use the difference between these last two m.d.i.s. to test these conditional probabilities, that is, the probability that an observation falls in the jth column given that it is in the ith row. The difference is

$$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{f_{i\cdot}\dfrac{p_{ij}}{p_{i\cdot}}}, \tag{3.4}$$

which is asymptotically distributed as χ^2 with $(rc-1)-(r-1)=r(c-1)$ degrees of freedom, that is, a test of r distributions of c categories each. A corresponding analysis can be made by interchanging i and j. The m.d.i.s. for testing column marginal probabilities $p_{\cdot j}$ is

$$2\sum_{j=1}^{c}f_{\cdot j}\ln\frac{f_{\cdot j}}{np_{\cdot j}}, \tag{3.5}$$

which is asymptotically distributed as χ^2 with $c-1$ degrees of freedom. Now let us suppose that the null hypothesis specifies that the row and column classifications are independent of each other in the probability sense; namely, that $p_{ij}=p_{i\cdot}p_{\cdot j}$. If this is so, then by subtracting the m.d.i.s. for columns in (3.5) from the value in (3.4), we obtain the m.d.i.s.

$$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{\dfrac{f_{i\cdot}f_{\cdot j}}{n}}. \tag{3.6}$$

This is the $2\hat{I}$ given in (3.1) for the contingency table, is the independence component of the total information, and is asymptotically distributed as χ^2 with $r(c-1)-(c-1)=(r-1)(c-1)$ degrees of freedom.

We summarize the preceding discussion and analysis in table 3.2.

TABLE 3.2

Component due to—	Information	D.F.
Total	$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{np_{ij}}$	$rc-1$
Rows	$2\sum_{i=1}^{r}f_{i\cdot}\ln\frac{f_{i\cdot}}{np_{i\cdot}}$	$r-1$
Conditional (total less rows)	$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{f_{i\cdot}\frac{p_{ij}}{p_{i\cdot}}}$	$r(c-1)$
For $p_{ij}=p_{i\cdot}p_{\cdot j}$,		
Columns	$2\sum_{j=1}^{c}f_{\cdot j}\ln\frac{f_{\cdot j}}{np_{\cdot j}}$	$c-1$
Independence (conditional less columns)	$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{\frac{f_{i\cdot}f_{\cdot j}}{n}}$	$(r-1)(c-1)$

Because of the properties of the logarithm and the convexity of the information function, we have that the various information components are non-negative, their degrees of freedom are additive, that

$$2\sum_{i=1}^{r}\sum_{j=1}^{c}f_{ij}\ln\frac{f_{ij}}{np_{i\cdot}p_{\cdot j}}\geqq 2\sum_{i=1}^{r}f_{i\cdot}\ln\frac{f_{i\cdot}}{np_{i\cdot}} \tag{3.7}$$

and similar inequalities. Thus the right-hand side in (3.7) is the minimum value of the left-hand side for grouping the rows over the columns.

We now illustrate the procedure with data from an unpublished preliminary report "A study of child behavior in relation to safety release devices for refrigerators," U.S. Department of Commerce, National Bureau of Standards. In an experiment,

the safety features of six devices were investigated. The subjects were 201 children of both sexes ranging in age from 2 to 5 years. The result for each child was classified either as a success or failure. It was suspected that the results might depend on the upbringing of the child, or the socioeconomic classification of the parents. The total number of years of education of the parents was used as a measure of socioeconomic status, and the results are summarized in table 3.3.

TABLE 3.3

Results	Parents' education, years				
	16-25	26-30	31-35	36-40	Total
Success	31	35	24	7	97
Failure	14	41	43	6	104
Total	45	76	67	13	201

We find that $2(201 \ln 201)=2131.929$, $2\Sigma\Sigma f_{ij} \ln f_{ij}=1364.942$, $2\Sigma f_{i.} \ln f_{i.}=1853.527$, $2\Sigma f_{.j} \ln f_{.j}=1630.989$, and thus according to (3.1), $2\hat{I}=1364.942+2131.929-1853.527-1630.989=12.355$. The classical χ^2 statistic yielded 12.13. Since the probability of exceeding the observed value of $2\hat{I}$ with 3 degrees of freedom is about 0.007, we may conclude that the results are dependent on the socioeconomic status of the parents.

A similar analysis may be used to test the homogeneity of two or more samples of one-way tables. In considering the various criteria of classification of a contingency table we must distinguish between those which are random variables and those which are nonrandom variables. The latter are those over which we may test for homogeneity. Statistical independence is a property of random variables. Both criteria of classification in table 3.3 are random variables since they were observed as characteristics of a child selected at random. Had the children been selected according to the parents' socioeconomic status then we would have been concerned with the homogeneity of the results over the statuses.

Let us suppose that we have r independent samples consisting of $n_1, n_2, \ldots, n_r$ independent observations, respectively, each sample having been randomly drawn from some c-valued population whose theoretical probabilities are unknown. The c-valued population may be either a discrete distribution of c categories or a continuous distribution that has been grouped into c classes. Let the observed frequencies for the ith sample ($i=1,2, \ldots, r$) be given by $f_{i1}, f_{i2}, \ldots, f_{ic}$, where c is the number of categories involved. We have

$$\sum_{j=1}^{c} f_{ij}=n_i, \text{ for } i=1, 2, \ldots, r, r\geqq 2, c\geqq 2;$$

$$\sum_{i=1}^{r} n_i=n; \sum_{i=1}^{r} f_{ij}=f_j$$

To test a null hypothesis that the samples are homogeneous, that is, are from the same, but unspecified population, against the alternative hypothesis that at least one pair of samples is from different populations, we use the m.d.i.s.

$$2\hat{I}=2 \sum_{i=1}^{r} \sum_{j=1}^{c} f_{ij} \ln f_{ij}+2n \ln n-2 \sum_{j=1}^{c} f_j \ln f_j -2 \sum_{i=1}^{r} n_i \ln n_i \quad (3.8)$$

The statistic in (3.8) is asymptotically distributed as χ^2 with $(r-1)(c-1)$ degrees of freedom under the null hypothesis that the r samples are from the same, but unspecified, population.

The analysis for the test of homogeneity, including the case when the hypothesis specifies the common population probabilities, is summarized in table 3.4, which is similar to table 3.2. In fact, the test statistics for independence and homogeneity are the same.

TABLE 3.4

Component due to—	Information	D.F.
Total	$2 \sum_{i=1}^{r} \sum_{j=1}^{c} f_{ij} \ln \frac{f_{ij}}{n_i p_j}$	$r(c-1)$
Specified p's	$2 \sum_{j=1}^{c} f_j \ln \frac{f_j}{np_j}$	$c-1$
Homogeneity	$2 \sum_{i=1}^{r} \sum_{j=1}^{c} f_{ij} \ln \frac{nf_{ij}}{n_i f_j}$	$(r-1)(c-1)$

An illustration of the use of (3.8) for the case of two samples may be found in Kupperman (1960). We illustrate the use of (3.8) here using data from the experiment described for table 3.3. In conducting the experiment care had been taken to assign to each device approximately the same number of children of the same age group for each sex. Thus, these two background variables were "balanced" so that any difference in the results could not be attributed to the bias, if any, introduced by age and sex. For our conclusion to be valid in the test above for independence between results and parents' education, it will be necessary to examine also the distribution of parents' education among the devices. In other words, are the distributions of parents' education among the six devices, as given in table 3.5, homogeneous?

TABLE 3.5

Device	Parents' education				
	16-25	26-30	31-35	36-40	Total
1	4	13	13	1	31
2	6	14	8	4	32
3	4	16	16	5	41
4	15	14	18	3	50
5	10	6	0	0	16
6	6	13	12	0	31
Total	45	76	67	13	201

We find that $2\Sigma\Sigma f_{ij} \ln f_{ij}=970.016$, $2\Sigma n_i \ln n_i=1432.059$, $2\Sigma f_j \ln f_j=1630.989$, and thus according to (3.8) $2\hat{I}=970.016+2131.929-1630.989-1432.059=$

38.897. The classical χ^2 statistic yielded 34.05. Since the probability of exceeding the observed value of $2\hat{I}$ with 15 degrees of freedom is about 0.0007, we conclude that the distribution of parents' education among the six devices is not homogeneous, and therefore further analysis or experimentation is necessary before we finally conclude that the results, success or failure, are dependent on parents' education.

It is evident from the above example, that the addition or deletion of classes in one or the other classifications will only alter the marginal and grand totals, and further analysis could be made with few additional computations. This is illustrated in the following example.

An instrument was designed to count the number of glass beads exceeding a certain diameter flowing between two electrodes. The beads were extremely small, having diameters between 2–70 microns, and were suspended in liquid. Three samples were measured by the instrument with the results shown in table 3.6. The homogeneity of the distribution of bead sizes in these samples is to be tested.

Table 3.6

Class No.	Bead size	Sample No.			
		201	202	203	Total
1	29.7 micron$<d$	62	71	82	215
2	25.9–29.7	102	107	123	332
3	20.6–25.9	892	1033	1122	3047
4	16.3–20.6	2503	2819	3071	8393
5	13.0–16.3	2480	2959	3115	8554
6	10.3–13.0	2149	2385	2558	7092
7	8.2–10.3	1740	1844	2105	5689
8	6.6–8.2	1673	1588	1707	4968
9	5.3–6.6	1190	1515	1451	4156
10	4.3–5.3	1070	1229	1121	3420
11	3.6–4.3	569	568	432	1569
		14430	16118	16887	47435

By using (3.8), the value of m.d.i.s. is easily (table look-up, addition, and subtraction) computed to be 123.47 with 20 degrees of freedom. The null hypothesis of homogeneity of the samples is therefore rejected.

However, the same samples were found to be homogeneous in their size-number distribution from results of microscopic measurements and counts, and the discrepancy was thought to be caused by insensitivity of the instrument in registering and differentiating beads below a certain size. This suggests the procedure of grouping the classes for the smaller sizes and testing the hypothesis of homogeneity of the three samples for the reduced number of classes. Classes 10 and 11 were grouped together first as one class, giving a value of m.d.i.s. of 105.758. Classes 9, 10, and 11 were then grouped and the value of m.d.i.s. calculated. The grouping procedure was continued and the results are shown in table 3.7.

Assuming that the microscopic method is the more reliable (though more tedious) one of the two tests, the analysis suggests that the instrument did not give reliable counts for beads with diameters less than 16.3 microns.

Table 3.7

Homogeneity of 3 samples	Information	D.F.	Probability
11 classes	123.47	20	<0.001
1,2, . . ., 9,(10+11)	105.76	18	<.001
1,2, . . ., 8,(9+10+11)	82.67	16	<.001
1,2, . . ., 7,(8+9+ . . . +11)	56.98	14	<.001
1,2, . . ., 6,(7+8+ . . . +11)	33.50	12	<.001
1,2, . . ., 5,(6+7+ . . . +11)	25.30	10	<.005
1,2, . . ., 4,(5+6+ . . . +11)	9.60	8	≈.25

To illustrate the partition of a contingency table into subcontingency tables and the test of homogeneity within and between subtables, we shall use the data in table 3.8.

In designing an acceptance sampling plan for clinical thermometers, certain lots of the total 61,000 thermometers were 100 percent inspected according to a threefold classification of defects: external physical defects "P," mercury column defects "M," and calibration defects "C." Manufacturer D delivered lots 1 to 4 at one time and lots 5 to 8 at a later date. One of the problems in the preliminary investigations was to determine whether the distribution of defects of thermometers between lots within a delivery were the same; also, whether these distributions remained unchanged between deliveries within a reasonable period of time. In the inspection process, if a thermometer was found to have "P" defects, it was set aside, and no further examination for "M" and "C" defects were made, etc. Thus, the three classifications were not necessarily mutually exclusive and the total number of individual defects may exceed the number of defects listed. However, since the test was made only for exploratory purpose, it was assumed that the number of thermometers having more than one type of defect is small and would not materially change the results of the analysis.

Table 3.8

Lots	Defects			No defects	Total
	P	M	C		
Group A:					
1	156	40	21	1030	1247
2	146	24	19	1054	1243
3	180	39	17	1011	1247
4	162	21	18	965	1166
Group B:					
5	91	47	10	1188	1336
6	114	28	11	1083	1236
7	103	39	13	1065	1220
8	100	31	9	992	1132
Total	1052	269	118	8388	9827

The analysis of information for the data in table 3.8 is given in table 3.9 (cf. Kullback (1959, pp. 173–176)), where n^A, n^B, f_j^A, f_j^B are, respectively, the totals and marginal column totals for groups A and B, that is

$$n^A=n_1+n_2+\ .\ .\ .\ +n_4,\ n^B=n_5+\ .\ .\ .\ +n_8,$$
$$f_j^A=f_{1j}+f_{2j}+\ .\ .\ .\ +f_{4j},\ f_j^B=f_{5j}+\ .\ .\ .\ +f_{8j},$$
$$n=n^A+n^B,\ f_j=f_j^A+f_j^B.$$

TABLE 3.9

Component due to—	Information	D.F.
Homogeneity, total	$2\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}\ln\frac{nf_{ij}}{n_i f_j}$	21
Homogeneity within group A ($i=1, \ldots, 4$)	$2\sum_{i=1}^{4}\sum_{j=1}^{4} f_{ij}\ln\frac{n^A f_{ij}}{n_i f_j^A}$	9
Homogeneity within group B ($i=5, \ldots, 8$)	$2\sum_{i=5}^{8}\sum_{j=1}^{4} f_{ij}\ln\frac{n^B f_{ij}}{n_i f_j^B}$	9
Homogeneity between groups A and B	$2\sum_{\alpha=A}^{B}\sum_{j=1}^{4} f_j^\alpha\ln\frac{nf_j^\alpha}{n^\alpha f_j}$	3

We find that $2(9827 \ln 9827)=180677.040$, $2(4924 \ln 4924)=83726.480$, $2(4903 \ln 4903)=83327.490$,

$$2\sum_{\alpha=A}^{B} n^\alpha \ln n^\alpha=167053.970,$$

$$2\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}\ln f_{ij}=129588.725,$$

$$2\sum_{i=1}^{4}\sum_{j=1}^{4} f_{ij}\ln f_{ij}=64066.128,$$

$$2\sum_{i=5}^{8}\sum_{j=1}^{4} f_{ij}\ln f_{ij}=65522.597,$$

$$2\sum_{i=1}^{8} n_i \ln n_i=139828.691,$$

$$2\sum_{i=1}^{4} n_i \ln n_i=69737.424,$$

$$2\sum_{i=5}^{8} n_i \ln n_i=70091.267,$$

$$2\sum_{j=1}^{4} f_j \ln f_j=170340.147,$$

$$2\sum_{j=1}^{4} f_j^A \ln f_j^A=77642.028,$$

$$2\sum_{j=1}^{4} f_j^B \ln f_j^B=79147.393,$$

$$2\sum_{\alpha=A}^{B}\sum_{j=1}^{4} f_j^\alpha \ln f_j^\alpha=156789.421.$$

We remind the reader that all the information components are sums and differences of the numerical values listed above. An advantage that the m.d.i.s. has over χ^2 statistics in such examples is that new samples can be added, or old ones removed, with few extra calculations. This advantage is most marked if we are trying iteratively to separate our samples into distinct homogeneous subsets.

The results of our computation are summarized in table 3.10.

TABLE 3.10

Component due to—	Information	D.F.
Homogeneity, total	96.927	21
Homogeneity, within group A	14.166	9
Homogeneity, within group B	10.417	9
Homogeneity, between group A and group B	72.344	3

We may conclude that, with respect to the distribution of defects, the eight lots were not homogeneous, that the first four lots of the first delivery were homogeneous, that the second four lots of the second delivery were homogeneous, and that the first delivery and the second delivery were not homogeneous.

For two-way contingency tables with the same number of rows and columns arising from related classifications, it is often of interest to test a null hypothesis of symmetry: that the events in cells symmetrically situated about the main diagonal have the same probability of occurrence. If p_{ij} is the probability of occurrence of the event in the ith row and jth column, then the null hypothesis is that $p_{ij}=p_{ji}$ for $i=1,2, \ldots, r$, $j=1,2, \ldots, r$, where r is the number of rows and also the number of columns.

If the probabilities p_{ij} are specified, then the m.d.i.s.

$$2\hat{I}=2\sum_{i=1}^{r}\sum_{j=1}^{r} f_{ij}\ln\frac{f_{ij}}{np_{ij}}, \text{ where } p_{ij}=p_{ji}, \qquad (3.9)$$

is asymptotically distributed as χ^2 with r^2-1 degrees of freedom if the null hypothesis that the probabilities are as specified is true.

To test the null hypothesis of symmetry without specified p_{ij} we use the m.d.i.s.

$$2\hat{I}=2\sum_{\substack{i=1\\ i\neq j}}^{r}\sum_{j=1}^{r} f_{ij}\ln\frac{2f_{ij}}{f_{ij}+f_{ji}}, \qquad (3.10)$$

which is asymptotically distributed as χ^2 with $\frac{r(r-1)}{2}$ degrees of freedom under the null hypothesis of symmetry.

An illustration of the application of (3.10) is given in Kullback (1959, pp. 179–180).

4. Three-Way and Higher-Order Tables

We shall characterize each observation by three criteria of classification: row, column, and depth, with r row categories, c column categories, and d depth categories, respectively.

The number of hypotheses of interest that may be tested in the case of three-way contingency tables is obviously greater than for the two-way contingency table. The statistical analysis proceeds, however, in a fashion similar to that developed in sections 2 and 3, the basic procedure being that given in section 2 for sampling from a c-valued population. In the present case the population has rcd classes, and by specializing hypotheses we use the results of sections 2 and 3 to obtain the appropriate m.d.i.s. The properties of the logarithm and convexity will assure that the m.d.i.s. and number of degrees of freedom are additive.

Let f_{ijk} be the frequency of occurrence of the observations in the cell in the ith row, jth column, and kth depth. We adopt the following notation:

$$\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{d} f_{ijk}=n,\ f_{i..}=\sum_{j=1}^{c}\sum_{k=1}^{d} f_{ijk},$$

$$f_{.j.}=\sum_{i=1}^{r}\sum_{k=1}^{d} f_{ijk},\ f_{..k}=\sum_{i=1}^{r}\sum_{j=1}^{c} f_{ijk},\ f_{ij.}=\sum_{k=1}^{d} f_{ijk},$$

$$f_{i.k}=\sum_{j=1}^{c} f_{ijk},\ f_{.jk}=\sum_{i=1}^{r} f_{ijk},$$

$$\sum_{i=1}^{r}\sum_{j=1}^{c} f_{ij.}=\sum_{i=1}^{r}\sum_{k=1}^{d} f_{i.k}=\sum_{j=1}^{c}\sum_{k=1}^{d} f_{.jk}=n,$$

$$\sum_{i=1}^{r} f_{i..}=\sum_{j=1}^{c} f_{.j.}=\sum_{k=1}^{d} f_{..k}=n.$$

The theoretical probabilities for the various cell entries and marginal totals are given by p's with corresponding subscripts.

One of the most important null hypotheses in connection with a three-way contingency table is that the three classifications (row, column, and depth) are independent. This null hypothesis is that $p_{ijk}=p_{i..}p_{.j.}p_{..k}$. The total m.d.i.s. is

$$2\hat{I}=\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{d} f_{ijk} \ln \frac{f_{ijk}}{np_{ijk}}, \tag{4.1}$$

which is asymptotically distributed as χ^2 with $(rcd-1)$ degrees of freedom under the null hypothesis that the cell probabilities are as specified by p_{ijk}. If the hypothesis specifies the marginal probabilities and that the row, column, and depth classifications are independent, then the statistic is

$$2\hat{I}=2\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{d} f_{ijk} \ln \frac{f_{ijk}}{np_{i..}p_{.j.}p_{..k}}, \tag{4.2}$$

which is asymptotically distributed as χ^2 with $(rcd-1)$ degrees of freedom. If the row marginal probabilities are specified as $p_{i..}$, then the m.d.i.s.

$$2\sum_{i=1}^{r} f_{i..} \ln \frac{f_{i..}}{np_{i..}} \tag{4.3}$$

is asymptotically distributed as χ^2 with $(r-1)$ degrees of freedom. And similarly for the other marginal probabilities:

$$\text{Columns: } 2\sum_{j=1}^{c} f_{.j.} \ln \frac{f_{.j.}}{np_{.j.}}, \qquad c-1 \text{ D.F.}, \tag{4.4}$$

$$\text{Depths: } 2\sum_{k=1}^{d} f_{..k} \ln \frac{f_{..k}}{np_{..k}}, \qquad d-1 \text{ D.F.} \tag{4.5}$$

Because of additivity (and the convexity property), we may subtract the sum of the last three information values from the total information to obtain

$$2\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{d} f_{ijk} \ln \frac{n^2 f_{ijk}}{f_{i..}f_{.j.}f_{..k}}, \tag{4.6}$$

which is the independence component when the marginal probabilities are not specified; this statistic is asymptotically distributed as χ^2 with $(rcd-r-c-d+2)$ degrees of freedom.

We summarize the preceding discussion in the analysis-of-information table 4.1 (cf. table 3.2).

Table 4.1

Component due to—	Information	D.F.
Total	$2\sum_i\sum_j\sum_k f_{ijk} \ln \frac{f_{ijk}}{np_{ijk}}$	$rcd-1$
Rows	$2\sum_i f_{i..} \ln \frac{f_{i..}}{np_{i..}}$	$r-1$
Conditional (given row totals)	$2\sum_i\sum_j\sum_k f_{ijk} \ln \frac{f_{ijk}}{f_{i..}\frac{p_{ijk}}{p_{i..}}}$	$r(cd-1)$
For $p_{ijk}=p_{i..}p_{.jk}$		
Columns	$2\sum_j f_{.j.} \ln \frac{f_{.j.}}{np_{.j.}}$	$c-1$
Conditional (given row and column totals).	$2\sum_i\sum_j\sum_k f_{ijk} \ln \frac{nf_{ijk}}{f_{i..}f_{.j.}\frac{p_{.jk}}{p_{.j.}}}$	$rcd-r-c+1$
Conditional (depth given column).	$2\sum_j\sum_k f_{.jk} \ln \frac{f_{.jk}}{f_{.j.}\frac{p_{.jk}}{p_{.j.}}}$	$c(d-1)$
Independence (row $\times$ (column, depth)).	$2\sum_i\sum_j\sum_k f_{ijk} \ln \frac{nf_{ijk}}{f_{i..}f_{.jk}}$	$(r-1)(cd-1)$
For $p_{.jk}=p_{.j.}p_{..k}$ in conditional (given row and column totals)		
Depths	$2\sum_k f_{..k} \ln \frac{f_{..k}}{np_{..k}}$	$d-1$
Independence	$2\sum_i\sum_j\sum_k f_{ijk} \ln \frac{n^2 f_{ijk}}{f_{i..}f_{.j.}f_{..k}}$	$rcd-r-c-d+2$

Many further analyses can be made of data categorized by three criteria of classification. We shall mention only a few of the possible analyses here and refer the reader to chapter 8 in Kullback (1959), where the subject is treated in detail. It is felt by the authors that these procedures should prove to be a useful practical statistical tool now that tables of $n \ln n$ and $2n \ln n$ are available.

We could test to see whether one classification is independent of the other classifications. This is an analogue of the multiple correlation procedure. There are three such tests possible, of course. For instance, the conditional (given row and column totals) term in table 4.1 has been analyzed into two additive terms, with respective degrees of freedom $c(d-1)$ and $(r-1)(cd-1)$. The independence row $\times$ (column, depth) component is the m.d.i.s. for testing that the row classification is independent of the other two classifications.

We can test for conditional independence, which means that for some particular category, say the kth, of the depth classification we may test a null hypothesis that the row and column classifications are independent. The procedure here is exactly as given in section 3 for the two-way contingency table. By summing with respect to k, we then get an analog of partial correlation.

If we have r independent samples of a two-way contingency table $(c\times d)$, we may treat the data as

an $r \times c \times d$ contingency table (three-way) with suitable hypotheses and restrictions. If we have rc independent samples of a one-way contingency table (d categories), we may again treat the data as an $r \times c \times d$ contingency table.

Suppose that we wish to test a null hypothesis that the r samples of a $c \times d$ contingency table are homogeneous, subject to a fixed total for each $c \times d$ table. The analysis is similar to that in table 3.4, where to test the homogeneity of r independent samples from a discrete population of cd distinct categories we set

$$j=(j, k),\ j=1, 2, \ldots, c,\ k=1, 2, \ldots, d.$$

The m.d.i.s. for the test of homogeneity is, therefore

$$2 \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{k=1}^{d} f_{ijk} \ln \frac{n f_{ijk}}{f_{i..} f_{.jk}}$$

with $(r-1)(cd-1)$ degrees of freedom. We remark that the m.d.i.s. for this test is identical to the m.d.i.s. for the test of independence between the row and the other two classifications; the interpretations of the results of these tests, however, are different.

As an example of the problem of conditional homogeneity, suppose that we have r independent samples of a $c \times d$ contingency table, then for some category, say the jth, of the column classification we could test a null hypothesis that the r samples of the depth classification are homogeneous. By summing over all j we obtain the m.d.i.s. for the test of conditional homogeneity of the depth classification for r samples, given the column classification. The analysis is given in table 4.2.

TABLE 4.2

Component due to—	Information	D.F.
Col., depth homogeneity (over r samples).	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{n f_{ijk}}{f_{i..} f_{.jk}}$	$(r-1)(cd-1)$
Col. homogeneity	$2 \sum_i \sum_j f_{ij.} \ln \frac{n f_{ij.}}{f_{i..} f_{.j.}}$	$(r-1)(c-1)$
Conditional homogeneity, depth given col.	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{(f_{ij.} f_{.jk})/f_{.j.}}$	$c(r-1)(d-1)$

We may carry the analogy between the analysis of information and the analysis of variance one step further and consider the conditional homogeneity component to be the sum of two components: one corresponding to the interaction of the depth classification with the row classification, and the other the interaction of the row-depth classification with the column classification. If we let

$$y_{i.k} = \sum_{j=1}^{c} f_{ij.} f_{.jk} / f_{.j.},$$

then

$$2 \sum_i \sum_k f_{i.k} \ln \frac{f_{i.k}}{y_{i.k}}$$

is the m.d.i.s. for the row-depth interaction. Since by (1.5)

$$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{(f_{ij.} f_{.jk})/f_{.j.}} \geq \sum_i \sum_k f_{i.k} \ln \frac{f_{i.k}}{y_{i.k}},$$

the difference is

$$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{f_{ij.} f_{.jk}}{f_{.j.}} \cdot \frac{f_{i.k}}{y_{i.k}}},$$

which is the m.d.i.s. for the interaction between the row-depth and column. The analysis is given in table 4.3.

TABLE 4.3

Component due to—	Information	D.F.
Conditional homogeneity, depth given col.	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{(f_{ij.} f_{.jk})/f_{.j.}}$	$c(r-1)(d-1)$
Row interaction with depth	$2 \sum_i \sum_k f_{i.k} \ln \frac{f_{i.k}}{y_{i.k}}$	$(r-1)(d-1)$
Row-depth interaction with column.	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{f_{ij.} f_{.jk}}{f_{.j.}} \cdot \frac{f_{i.k}}{y_{i.k}}}$	$(r-1)(c-1)(d-1)$

Alternatively, it is also possible to analyze the conditional homogeneity component in table 4.3 algebraically into two other components; namely, the homogeneity component of depth (with row) and the three-factor interaction component among row, column, and depth as shown in table 4.4.

TABLE 4.4

Component due to—	Information	D.F.
Conditional homogeneity, depth given col.	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{(f_{ij.} f_{.jk})/f_{.j.}}$	$c(r-1)(d-1)$
Depth homogeneity	$2 \sum_i \sum_k f_{i.k} \ln \frac{n f_{i.k}}{f_{i..} f_{..k}}$	$(r-1)(d-1)$
Three-factor interaction	$2 \sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{n f_{ij.} f_{i.k} f_{.jk}}{f_{i..} f_{.j.} f_{..k}}}$	

This three-factor interaction component may also be derived from the row-depth by column interaction component in table 4.3. Note that the row-depth interaction with column component will become the three-factor interaction component if the columns are homogeneous over the rows, that is, if $f_{ij.} = \frac{f_{i..} f_{.j.}}{n}$. Then

$$y_{i.k} = \sum_j \frac{f_{ij.} f_{.jk}}{f_{.j.}} = \sum_j \frac{f_{i..} f_{.jk}}{n} = \frac{f_{i..} f_{..k}}{n}$$

and the three-factor interaction component in table 4.4 results. However, since the convexity property

is not used here, there is no guarantee that the conditional homogeneity component in table 4.4 is in fact larger than the depth homogeneity component. Therefore, the three-factor interaction component computed may turn out to be negative. In such cases the analysis in table 4.3 is the proper one to follow.

The three-factor interaction problem has been investigated by Bartlett (1935), Norton (1945), Kastenbaum and Lamphiear (1959), Roy and Mitra (1956), Plackett (1962), and Darroch (1962).

Kastenbaum and Lamphiear (1959) used the data shown in table 4.5 and concluded that the hypothesis of no three-factor interaction is tenable by an estimation process involving the solution of $(r-1)(c-1)(d-1)$ simultaneous third-degree equations in as many unknowns.

TABLE 4.5

Litter sizes	7		8		9		10		11		Total
Treatment	A	B	A	B	A	B	A	B	A	B	
Number of depletions:											
0	58	75	49	58	33	45	15	39	4	5	381
1	11	19	14	17	18	22	13	22	12	15	163
2 or more	5	7	10	8	15	10	15	18	17	8	113
Total	74	101	73	83	66	77	43	79	33	28	657

"The data represent a portion of an experiment performed at the Oak Ridge National Laboratory. For this experiment litters of mice of various sizes were treated in either one of two different ways, and the number of deaths per litter before weaning observed.

"For each of the two treatments a chi-square statistic with 8 degrees of freedom was computed to test the hypothesis that the number of depletions is independent of litter size. This hypothesis was rejected for both treatments. In addition, for each of the five litter sizes, chi-square with 2 degrees of freedom was computed to test the hypothesis that the number of depletions is independent of the treatments. In each case, there was no reason to reject this hypothesis. From these two sets of tests, one may conclude first that the number of depletions is dependent upon litter sizes for both treatment A and treatment B, and, secondly, that the treatments do not significantly affect the number of depletions in any of the litter sizes.

"The 'no-interaction' chi-square confirms these results. This chi-square, with 8 degrees of freedom, is found to be 3.158 and corresponds to a probability of greater than 90 per cent. From this result we may conclude that the $2\times3\times5$ table is homogeneous. That is to say, the significant interaction between the number of depletions and the size of litter is consistent for both treatments and the lack of interaction between the number of depletions and treatment is consistent for all litter sizes."

We shall designate treatment as the row R, depletion as the column C, and litter size as the depth D classifications, and form the three marginal two-way tables 4.6, 4.7, and 4.8 ($i=1,2$, $j=1,2,3$, $k=1,2,3,4,5$).

TABLE 4.6

R \ C	0	1	2 or more	
A	159	68	62	289
B	222	95	51	368
	381	163	113	657

TABLE 4.7

R \ D	7	8	9	10	11	
A	74	73	66	43	33	289
B	101	83	77	79	28	368
	175	156	143	122	61	657

TABLE 4.8

C \ D	7	8	9	10	11	
0	133	107	78	54	9	381
1	30	31	40	35	27	163
2 or more	12	18	25	33	25	113
	175	156	143	122	61	657

We find that:

$$2\,n \ln n=8524.817,$$

$$2 \sum_i f_{i..} \ln f_{i..}=7623.544,\quad 2 \sum_j f_{.j.} \ln f_{.j.}=6476.312,$$

$$2 \sum_k f_{..k} \ln f_{..k}=7257.366,$$

$$2 \sum_i \sum_j f_{ij.} \ln f_{ij.}=6362.602,\quad 2 \sum_i \sum_k f_{i.k} \ln f_{i.k}$$

$$=5582.380,\quad 2 \sum_j \sum_k f_{.jk} \ln f_{.jk}=5306.112,$$

$$2 \sum_i \sum_j \sum_k f_{ijk} \ln f_{ijk}=4421.946,$$

$$2\, f_{1jk} \ln f_{1jk}=1853.498,\quad 2\, f_{2jk} \ln f_{2jk}=2568.448,$$

$$2\, f_{1j.} \ln f_{1j.}=2697.530,\quad 2\, f_{2j.} \ln f_{2j.}=3665.072,$$

$$2\, f_{1.k} \ln f_{1.k}=2370.676,\quad 2\, f_{2.k} \ln f_{2.k}=3211.704,$$

$$2\, f_{1..} \ln f_{1..}=3275.195,\quad 2\, f_{2..} \ln f_{2..}=4348.349.$$

The analyses are shown in tables 4.9 and 4.10, and the relevant hypotheses tested. Although each m.d.i.s. component corresponds to a particular hypothesis for the three-way table, only those of interest to the problem and those with a valid physical interpretation are so identified.

H_1: The three classifications are independent. *H_1 is rejected.*

H_2: The number of depletions is independent of the litter size. *H_2 is rejected.*

H_3: The number of depletions for each litter size is independent of treatment (or the number of depletions for each litter size is the same for treatment A and treatment B). *H_3 is accepted.*

H_4: Given the litter size, the number of depletions is independent of the treatment (this component can be further analyzed into 5 components with 2 degrees of freedom each for each litter size). *H_4 is accepted.*

H_5: There is no three-way interaction. *H_5 is accepted.*

H_6: Given treatment, the depletion is independent of litter size. (This component can be further analyzed into 2 components with 8 degrees of freedom each for each treatment.) *H_6 is rejected.*

Here we reach the same conclusions as those reached by Kastenbaum and Lamphiear. The analysis is straightforward and the computation can be easily performed with the help of the $2n \ln n$ table.

TABLE 4.9

Component due to—	Information	D.F.
H_1	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{n^2 f_{ijk}}{f_{i..}f_{i..}f_{.j.}f_{..k}}=114.358$	$rcd-r-c-d+2=22$
H_2	$2\sum_j \sum_k f_{.jk} \ln \frac{nf_{.jk}}{f_{.j.}f_{..k}}=97.251$	$(c-1)(d-1)=8$
H_3	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{nf_{ijk}}{f_{i..}f_{.jk}}=17.107$	$(r-1)(cd-1)=14$
	$2\sum_i \sum_k f_{i.k} \ln \frac{nf_{i.k}}{f_{i..}f_{..k}}=7.341$	$(r-1)(d-1)=4$
H_4	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{f_{i.k}f_{.jk}}{f_{..k}}}=9.766$	$d(r-1)(c-1)=10$
	$2\sum_i \sum_j f_{ij.} \ln \frac{nf_{ij.}}{f_{i..}f_{.j.}}=6.509$	$(r-1)(c-1)=2$
H_5	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{nf_{ij.}f_{i.k}f_{.jk}}{f_{i..}f_{.j}f_{..k}}}=3.257$	$(r-1)(c-1)(d-1)=8$

TABLE 4.10

Component due to—	Information	D.F.
H_1	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{n^2 f_{ijk}}{f_{i..}f_{.j.}f_{..k}}=114.358$	$rcd-r-c-d+2=22$
	$2\sum_i \sum_j f_{ij.} \ln \frac{nf_{ij.}}{f_{i..}f_{.j.}}=6.509$	$(r-1)(c-1)=2$
	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{nf_{ijk}}{f_{..k}f_{ij.}}=107.849$	$(rc-1)(d-1)=20$
	$2\sum_i \sum_k f_{i.k} \ln \frac{nf_{i.k}}{f_{i..}f_{..k}}=7.341$	$(r-1)(d-1)=4$
H_6	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{f_{ij.}f_{i.k}}{f_{i..}}}=100.508$	$r(c-1)(d-1)=16$
	$2\sum_j \sum_k f_{.jk} \ln \frac{nf_{.jk}}{f_{.j.}f_{..k}}=97.251$	$(c-1)(d-1)=8$
H_5	$2\sum_i \sum_j \sum_k f_{ijk} \ln \frac{f_{ijk}}{\frac{nf_{ij.}f_{i.k}f_{.jk}}{f_{i..}f_{.j.}f_{..k}}}=3.257$	$(r-1)(c-1)(d-1)=8$

It is of interest to compare the results of the analysis in accordance with the type of analysis in table 4.3. The values of $y_{.jk}$ computed in accordance with $y_{.jk}=\sum_{i=1}^{r} f_{ij.}f_{i.k}/f_{i..}$ are given in table 4.11.

TABLE 4.11

$y_{.jk}$

C \ D	7	8	9	10	11	
0	101.64	90.23	82.76	71.32	35.06	381.00
1	43.48	38.61	35.41	30.51	14.99	163.00
2 or more	29.88	27.16	24.83	20.17	10.96	113.00
	175.00	156.00	143.00	122.00	61.00	657.00

It is found that $2\sum_j \sum_k f_{.jk} \ln y_{.jk}=5209.244$.

The results of the analysis with the computed values of $y_{.jk}$ are given in table 4.12.

TABLE 4.12

Component due to—	Information	D.F.
H_6	100.508	16
CD—interaction (Column interaction with depth)	96.868	8
CD, R—interaction (Column—depth interaction with row)	3.640	8

The procedure of this and preceding sections may be extended to the analysis of data in the form of higher-order contingency tables, and would definitely be simpler in statistical practice than techniques using χ^2 statistics. For four-way contingency tables this has been done by Ku (1960). An example of a detailed analysis of a four-way table is given in Kullback, Kupperman, and Ku (1962).

5. Markov Chains

The statistical tests described thus far have all been derived on the assumption that the successive observations are independent in the probability sense. There frequently arise practical situations wherein this assumption is no longer valid, for example, observations resulting from realizations of the states of a simple stationary Markov chain. Statistical methods in Markov chains using χ^2 and Ψ^2 statistics have been developed. For a recent summary see Billingsley (1961).

If the successive pairs of observations of the occurrences of the states of a Markov chain are distributed in a two-way contingency table, with the first state of the pair as the row category and the second (following) state of the pair as the column category, then the number of entries in the ith row and the jth column represent the number of observed occurrences of . . . E_iE_j . . ., and $P\{E_j|E_i\}$ is the conditional probability specified by the transition probability matrix. Since the asymptotic multinomial behavior of a simple finite Markov chain is well known (see, for example, Bartlett (1955).

Rosenblatt (1962)), the analyses of the information components of the resulting square matrix formally parallel those of the contingency table discussions. We could, for instance, test

(a) the hypothesis of a specified matrix of transition probabilities,

(b) the hypothesis of an unspecified matrix of transition probabilities,

(c) the homogeneity of several realizations of Markov chains of order 1, and

(d) the hypothesis of Markovity of order $r \geq 2$.

Development of these tests together with worked examples are given in Kullback, Kupperman, and Ku (1962).

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7. Appendix

Table of 2n ln n *for values of* n *from 1 to 10,000*

	0	1	2	3	4	5	6	7	8	9	
0	0. 000	0. 000	2. 773	6. 592	11. 090	16. 094	21. 501	27. 243	33. 271	39. 550	0
1	46. 052	52. 754	59. 638	66. 689	73. 894	81. 242	88. 723	96. 329	104. 053	111. 889	1
2	119. 829	127. 870	136. 006	144. 233	152. 547	160. 944	169. 421	177. 975	186. 603	195. 303	2
3	204. 072	212. 907	221. 807	230. 770	239. 793	248. 874	258. 013	267. 208	276. 457	285. 758	3
4	295. 110	304. 513	313. 964	323. 463	333. 009	342. 600	352. 235	361. 914	371. 635	381. 398	4
5	391. 202	401. 046	410. 929	420. 851	430. 810	440. 807	450. 839	460. 908	471. 011	481. 149	5
6	491. 321	501. 527	511. 765	522. 035	532. 337	542. 670	553. 034	563. 429	573. 853	584. 307	6
7	594. 789	605. 301	615. 840	626. 407	637. 002	647. 623	658. 271	668. 946	679. 647	690. 373	7
8	701. 124	711. 901	722. 702	733. 528	744. 377	755. 251	766. 148	777. 068	788. 011	798. 977	8
9	809. 966	820. 976	832. 009	843. 064	854. 139	865. 237	876. 355	887. 494	898. 654	909. 834	9
10	921. 034	932. 254	943. 494	954. 754	966. 033	977. 332	988. 649	999. 985	1011. 340	1022. 714	10
11	1034. 106	1045. 516	1056. 944	1068. 390	1079. 853	1091. 334	1102. 833	1114. 349	1125. 882	1137. 431	11
12	1148. 998	1160. 581	1172. 181	1183. 797	1195. 430	1207. 078	1218. 743	1230. 424	1242. 120	1253. 832	12
13	1265. 559	1277. 302	1289. 060	1300. 833	1312. 621	1324. 424	1336. 242	1348. 075	1359. 922	1371. 784	13
14	1383. 660	1395. 550	1407. 455	1419. 374	1431. 306	1443. 253	1455. 213	1467. 187	1479. 175	1491. 176	14
15	1503. 191	1515. 219	1527. 260	1539. 314	1551. 381	1563. 462	1575. 555	1587. 661	1599. 780	1611. 912	15
16	1624. 056	1636. 212	1648. 381	1660. 563	1672. 756	1684. 962	1697. 180	1709. 410	1721. 652	1733. 906	16
17	1746. 171	1758. 449	1770. 738	1783. 039	1795. 351	1807. 675	1820. 010	1832. 357	1844. 715	1857. 084	17
18	1869. 464	1881. 856	1894. 258	1906. 672	1919. 096	1931. 532	1943. 978	1956. 435	1968. 902	1981. 380	18
19	1993. 869	2006. 368	2018. 878	2031. 398	2043. 929	2056. 470	2069. 021	2081. 582	2094. 154	2106. 735	19
20	2119. 327	2131. 929	2144. 540	2157. 162	2169. 793	2182. 434	2195. 085	2207. 746	2220. 416	2233. 096	20
21	2245. 785	2258. 484	2271. 193	2283. 910	2296. 638	2309. 374	2322. 120	2334. 875	2347. 640	2360. 413	21
22	2373. 196	2385. 988	2398. 789	2411. 599	2424. 417	2437. 245	2450. 082	2462. 927	2475. 782	2488. 645	22
23	2501. 516	2514. 397	2527. 286	2540. 184	2553. 090	2566. 005	2578. 929	2591. 861	2604. 801	2617. 750	23
24	2630. 707	2643. 672	2656. 646	2669. 628	2682. 618	2695. 617	2708. 623	2721. 638	2734. 661	2747. 692	24
25	2760. 730	2773. 777	2786. 832	2799. 895	2812. 966	2826. 044	2839. 131	2852. 225	2865. 327	2878. 437	25
26	2891. 554	2904. 680	2917. 813	2930. 953	2944. 101	2957. 257	2970. 420	2983. 591	2996. 769	3009. 955	26
27	3023. 148	3036. 348	3049. 556	3062. 772	3075. 994	3089. 224	3102. 461	3115. 706	3128. 957	3142. 216	27
28	3155. 482	3168. 755	3182. 036	3195. 323	3208. 617	3221. 919	3235. 227	3248. 543	3261. 865	3275. 195	28
29	3288. 531	3301. 874	3315. 224	3328. 581	3341. 945	3355. 315	3368. 693	3382. 077	3395. 468	3408. 865	29
30	3422. 269	3435. 680	3449.098	3462. 522	3475. 953	3489. 390	3502. 834	3516. 285	3529. 741	3543. 205	30
31	3556. 675	3570. 151	3583. 634	3597. 123	3610. 619	3624. 121	3637. 629	3651. 144	3664. 665	3678. 192	31
32	3691. 725	3705. 265	3718. 811	3732. 363	3745. 922	3759. 486	3773. 057	3786. 634	3800. 217	3813. 806	32
33	3827. 401	3841. 002	3854. 610	3868. 223	3881. 842	3895. 467	3909. 099	3922. 736	3936. 379	3950. 028	33
34	3963. 683	3977. 344	3991. 011	4004. 683	4018. 361	4032. 046	4045. 736	4059. 431	4073. 133	4086. 840	34
35	4100. 553	4114. 272	4127. 996	4141. 726	4155. 462	4169. 204	4182. 951	4196. 703	4210. 462	4224. 225	35
36	4237. 995	4251. 770	4265. 550	4279. 336	4293. 128	4306. 925	4320. 728	4334. 536	4348. 349	4362. 168	36
37	4375. 992	4389. 822	4403. 657	4417. 498	4431. 343	4445. 195	4459. 051	4472. 913	4486. 780	4500. 652	37
38	4514. 530	4528. 413	4542. 301	4556. 195	4570. 093	4583. 997	4597. 906	4611. 821	4625. 740	4639. 665	38
39	4653. 594	4667. 529	4681. 469	4695. 414	4709. 365	4723. 320	4737. 280	4751. 245	4765. 216	4779. 191	39
40	4793. 172	4807. 157	4821. 147	4835. 143	4849. 143	4863. 149	4877. 159	4891. 174	4905. 194	4919. 219	40
41	4933. 249	4947. 284	4961. 323	4975. 368	4989. 417	5003. 471	5017. 530	5031. 594	5045. 662	5059. 736	41
42	5073. 814	5087. 897	5101. 984	5116. 077	5130. 174	5144. 276	5158. 382	5172. 494	5186. 609	5200. 730	42
43	5214. 855	5228. 985	5243. 120	5257. 259	5271. 403	5285. 551	5299. 704	5313. 862	5328. 024	5342. 190	43
44	5356. 362	5370. 538	5384. 718	5398. 903	5413. 092	5427. 286	5441. 485	5455. 687	5469. 895	5484. 107	44
45	5498. 323	5512. 544	5526. 769	5540. 998	5555. 232	5569. 471	5583. 713	5597. 961	5612. 212	5626. 468	45
46	5640. 728	5654. 993	5669. 262	5683. 535	5697. 813	5712. 095	5726. 381	5740. 672	5754. 966	5769. 265	46
47	5783. 569	5797. 876	5812. 188	5826. 504	5840. 825	5855. 149	5869. 478	5883. 811	5898. 148	5912. 489	47
48	5926. 835	5941. 184	5955. 538	5969. 896	5984. 258	5998. 624	6012. 995	6027. 369	6041. 748	6056. 131	48
49	6070. 517	6084. 908	6099. 303	6113. 702	6128. 105	6142. 512	6156. 923	6171. 338	6185. 758	6200. 181	49
50	6214. 608	6229. 039	6243. 475	6257. 914	6272. 357	6286. 804	6301. 255	6315. 710	6330. 169	6344. 632	50
51	6359. 099	6373. 570	6388. 044	6402. 523	6417. 006	6431. 492	6445. 982	6460. 476	6474. 974	6489. 476	51
52	6503. 982	6518. 492	6533. 005	6547. 522	6562. 043	6576. 568	6591. 097	6605. 629	6620. 166	6634. 706	52
53	6649. 250	6663. 797	6678. 349	6692. 904	6707. 463	6722. 025	6736. 592	6751. 162	6765. 736	6780. 313	53
54	6794. 895	6809. 480	6824. 068	6838. 661	6853. 257	6867. 857	6882. 460	6897. 067	6911. 678	6926. 292	54
55	6940. 910	6955. 532	6970. 157	6984. 786	6999. 418	7014. 055	7028. 694	7043. 338	7057. 985	7072. 635	55
56	7087. 289	7101. 947	7116. 608	7131. 273	7145. 941	7160. 613	7175. 289	7189. 967	7204. 650	7219. 336	56
57	7234. 025	7248. 718	7263. 415	7278. 115	7292. 819	7307. 526	7322. 236	7336. 950	7351. 667	7366. 388	57
58	7381. 113	7395. 840	7410. 572	7425. 306	7440. 044	7454. 786	7469. 531	7484. 279	7499. 031	7513. 786	58
59	7528. 545	7543. 307	7558. 072	7572. 841	7587. 613	7602. 388	7617. 167	7631. 949	7646. 735	7661. 523	59
60	7676. 316	7691. 111	7705. 910	7720. 712	7735. 518	7750. 326	7765. 139	7779. 954	7794. 773	7809. 595	60
61	7824. 420	7839. 248	7854. 080	7868. 915	7883. 754	7898. 595	7913. 440	7928. 288	7943. 140	7957. 994	61
62	7972. 852	7987. 713	8002. 577	8017. 445	8032. 316	8047. 190	8062. 067	8076. 947	8091. 830	8106. 717	62
63	8121. 607	8136. 500	8151. 396	8166. 296	8181. 198	8196. 104	8211. 013	8225. 925	8240. 840	8255. 758	63
64	8270. 679	8285. 604	8300. 531	8315. 462	8330. 396	8345. 333	8360. 273	8375. 216	8390. 162	8405. 112	64
65	8420. 064	8435. 020	8449. 978	8464. 940	8479. 904	8494. 872	8509. 843	8524. 817	8539. 794	8554. 774	65
66	8569. 757	8584. 743	8599. 732	8614. 724	8629. 719	8644. 717	8659. 718	8674. 722	8689. 729	8704. 739	66
67	8719. 752	8734. 768	8749. 787	8764. 809	8779. 834	8794. 862	8809. 893	8824. 927	8839. 964	8855. 003	67
68	8870. 046	8885. 092	8900. 140	8915. 192	8930. 246	8945. 304	8960. 364	8975. 427	8990. 493	9005. 562	68
69	9020. 634	9035. 709	9050. 787	9065. 868	9080. 951	9096. 037	9111. 127	9126. 219	9141. 314	9156. 412	69
70	9171. 512	9186. 616	9201. 722	9216. 832	9231. 944	9247. 059	9262. 177	9277. 297	9292. 421	9307. 547	70

Table of 2n ln n *for values of* n *from 1 to 10,000*—Continued

	0	1	2	3	4	5	6	7	8	9	
71	9322.676	9337.808	9352.943	9368.081	9383.221	9398.364	9413.510	9428.659	9443.810	9458.965	71
72	9474.122	9489.282	9504.444	9519.610	9534.778	9549.949	9565.123	9580.299	9595.478	9610.660	72
73	9625.845	9641.032	9656.223	9671.416	9686.611	9701.810	9717.011	9732.215	9747.421	9762.630	73
74	9777.842	9793.057	9808.274	9823.494	9838.717	9853.942	9869.171	9884.401	9899.635	9914.871	74
75	9930.110	9945.351	9960.595	9975.842	9991.092	10006.344	10021.599	10036.856	10052.116	10067.379	75
76	10082.644	10097.912	10113.183	10128.456	10143.732	10159.010	10174.291	10189.575	10204.861	10220.150	76
77	10235.441	10250.735	10266.032	10281.331	10296.633	10311.938	10327.245	10342.554	10357.866	10373.181	77
78	10388.499	10403.818	10419.141	10434.466	10449.793	10465.123	10480.456	10495.791	10511.129	10526.469	78
79	10541.812	10557.157	10572.505	10587.856	10603.209	10618.564	10633.922	10649.282	10664.645	10680.011	79
80	10695.379	10710.749	10726.122	10741.498	10756.876	10772.256	10787.639	10803.024	10818.412	10833.803	80
81	10849.195	10864.591	10879.989	10895.389	10910.791	10926.197	10941.604	10957.014	10972.427	10987.842	81
82	11003.259	11018.679	11034.101	11049.526	11064.953	11080.383	11095.815	11111.249	11126.686	11142.125	82
83	11157.567	11173.011	11188.457	11203.906	11219.357	11234.811	11250.267	11265.725	11281.186	11296.650	83
84	11312.115	11327.583	11343.054	11358.526	11374.001	11389.479	11404.959	11420.441	11435.926	11451.412	84
85	11466.902	11482.393	11497.887	11513.384	11528.882	11544.384	11559.887	11575.393	11590.901	11606.411	85
86	11621.924	11637.439	11652.956	11668.476	11683.998	11699.522	11715.049	11730.578	11746.109	11761.642	86
87	11777.178	11792.716	11808.257	11823.799	11839.344	11854.892	11870.441	11885.993	11901.547	11917.104	87
88	11932.663	11948.224	11963.787	11979.352	11994.920	12010.490	12026.062	12041.637	12057.214	12072.793	88
89	12088.374	12103.958	12119.544	12135.132	12150.722	12166.314	12181.909	12197.506	12213.105	12228.707	89
90	12244.311	12259.916	12275.525	12291.135	12306.747	12322.362	12337.979	12353.598	12369.220	12384.843	90
91	12400.469	12416.097	12431.727	12447.360	12462.994	12478.631	12494.270	12509.911	12525.554	12541.200	91
92	12556.848	12572.497	12588.149	12603.804	12619.460	12635.118	12650.779	12666.442	12682.107	12697.774	92
93	12713.443	12729.115	12744.788	12760.464	12776.142	12791.822	12807.504	12823.188	12838.875	12854.563	93
94	12870.254	12885.947	12901.642	12917.339	12933.038	12948.740	12964.443	12980.148	12995.856	13011.566	94
95	13027.278	13042.992	13058.708	13074.426	13090.146	13105.869	13121.593	13137.320	13153.048	13168.779	95
96	13184.512	13200.247	13215.984	13231.723	13247.464	13263.207	13278.953	13294.700	13310.449	13326.201	96
97	13341.954	13357.710	13373.468	13389.227	13404.989	13420.753	13436.519	13452.287	13468.057	13483.829	97
98	13499.603	13515.379	13531.157	13546.938	13562.720	13578.504	13594.290	13610.079	13625.869	13641.661	98
99	13657.456	13673.252	13689.051	13704.851	13720.654	13736.458	13752.265	13768.073	13783.884	13799.696	99
100	13815.511	13831.327	13847.146	13862.966	13878.789	13894.613	13910.440	13926.268	13942.098	13957.931	100
101	13973.765	13989.602	14005.440	14021.280	14037.123	14052.967	14068.813	14084.662	14100.512	14116.364	101
102	14132.218	14148.074	14163.932	14179.792	14195.654	14211.518	14227.384	14243.252	14259.122	14274.993	102
103	14290.867	14306.743	14322.620	14338.500	14354.381	14370.264	14386.150	14402.037	14417.926	14433.817	103
104	14449.710	14465.605	14481.502	14497.401	14513.301	14529.204	14545.108	14561.015	14576.923	14592.833	104
105	14608.745	14624.659	14640.575	14656.493	14672.413	14688.335	14704.258	14720.184	14736.111	14752.040	105
106	14767.971	14783.904	14799.839	14815.776	14831.715	14847.655	14863.597	14879.542	14895.488	14911.436	106
107	14927.386	14943.338	14959.291	14975.247	14991.204	15007.163	15023.124	15039.087	15055.052	15071.019	107
108	15086.987	15102.958	15118.930	15134.904	15150.880	15166.858	15182.837	15198.819	15214.802	15230.787	108
109	15246.774	15262.763	15278.753	15294.746	15310.740	15326.736	15342.734	15358.734	15374.735	15390.739	109
110	15406.744	15422.751	15438.760	15454.771	15470.783	15486.797	15502.813	15518.831	15534.851	15550.873	110
111	15566.896	15582.921	15598.948	15614.977	15631.007	15647.040	15663.074	15679.110	15695.147	15711.187	111
112	15727.228	15743.271	15759.316	15775.363	15791.411	15807.461	15823.513	15839.567	15855.622	15871.680	112
113	15887.739	15903.800	15919.862	15935.927	15951.993	15968.061	15984.130	16000.202	16016.275	16032.350	113
114	16048.426	16064.505	16080.585	16096.667	16112.751	16128.836	16144.923	16161.012	16177.103	16193.195	114
115	16209.290	16225.386	16241.483	16257.583	16273.684	16289.786	16305.891	16321.997	16338.105	16354.215	115
116	16370.327	16386.440	16402.555	16418.671	16434.790	16450.910	16467.032	16483.155	16499.281	16515.407	116
117	16531.536	16547.666	16563.799	16579.932	16596.068	16612.205	16628.344	16644.485	16660.627	16676.771	117
118	16692.917	16709.064	16725.213	16741.364	16757.516	16773.670	16789.826	16805.984	16822.143	16838.304	118
119	16854.466	16870.631	16886.797	16902.964	16919.134	16935.304	16951.477	16967.651	16983.827	17000.005	119
120	17016.184	17032.365	17048.548	17064.732	17080.918	17097.106	17113.295	17129.486	17145.679	17161.873	120
121	17178.069	17194.267	17210.466	17226.667	17242.869	17259.073	17275.279	17291.487	17307.696	17323.907	121
122	17340.119	17356.333	17372.549	17388.766	17404.985	17421.205	17437.428	17453.652	17469.877	17486.104	122
123	17502.333	17518.563	17534.795	17551.029	17567.264	17583.501	17599.739	17615.979	17632.221	17648.464	123
124	17664.709	17680.956	17697.204	17713.454	17729.705	17745.958	17762.213	17778.469	17794.727	17810.986	124
125	17827.247	17843.510	17859.774	17876.040	17892.307	17908.576	17924.847	17941.119	17957.393	17973.668	125
126	17989.945	18006.223	18022.503	18038.785	18055.068	18071.353	18087.640	18103.928	18120.217	18136.509	126
127	18152.801	18169.096	18185.392	18201.689	18217.988	18234.289	18250.591	18266.895	18283.200	18299.507	127
128	18315.815	18332.125	18348.437	18364.750	18381.065	18397.381	18413.699	18430.018	18446.339	18462.662	128
129	18478.986	18495.311	18511.638	18527.967	18544.297	18560.629	18576.962	18593.297	18609.633	18625.971	129
130	18642.311	18658.652	18674.994	18691.338	18707.684	18724.031	18740.380	18756.730	18773.082	18789.435	130
131	18805.790	18822.146	18838.504	18854.863	18871.224	18887.587	18903.951	18920.316	18936.683	18953.052	131
132	18969.422	18985.793	19002.166	19018.541	19034.917	19051.295	19067.674	19084.054	19100.436	19116.820	132
133	19133.205	19149.592	19165.980	19182.369	19198.761	19215.153	19231.547	19247.943	19264.340	19280.739	133
134	19297.139	19313.540	19329.943	19346.348	19362.754	19379.162	19395.571	19411.981	19428.393	19444.807	134
135	19461.222	19477.638	19494.056	19510.475	19526.896	19543.319	19559.743	19576.168	19592.595	19609.023	135
136	19625.453	19641.884	19658.317	19674.751	19691.186	19707.623	19724.062	19740.502	19756.944	19773.386	136
137	19789.831	19806.277	19822.724	19839.173	19855.623	19872.075	19888.528	19904.983	19921.439	19937.896	137
138	19954.355	19970.815	19987.277	20003.741	20020.205	20036.672	20053.139	20069.608	20086.079	20102.551	138
139	20119.024	20135.499	20151.975	20168.453	20184.932	20201.413	20217.895	20234.378	20250.863	20267.349	139
140	20283.837	20300.326	20316.817	20333.309	20349.802	20366.297	20382.793	20399.291	20415.790	20432.291	140

Table of 2n ln n *for values of* n *from 1 to 10,000*—Continued

	0	1	2	3	4	5	6	7	8	9	
141	20448.793	20465.296	20481.801	20498.307	20514.815	20531.324	20547.834	20564.346	20580.860	20597.374	141
142	20613.891	20630.408	20646.927	20663.447	20679.969	20696.492	20713.017	20729.543	20746.070	20762.599	142
143	20779.129	20795.661	20812.194	20828.728	20845.264	20861.801	20878.339	20894.879	20911.421	20927.963	143
144	20944.507	20961.053	20977.600	20994.148	21010.698	21027.249	21043.801	21060.355	21076.910	21093.467	144
145	21110.025	21126.584	21143.145	21159.707	21176.270	21192.835	21209.401	21225.969	21242.538	21259.108	145
146	21275.680	21292.253	21308.827	21325.403	21341.980	21358.559	21375.139	21391.720	21408.303	21424.887	146
147	21441.472	21458.059	21474.647	21491.236	21507.827	21524.419	21541.013	21557.607	21574.204	21590.801	147
148	21607.400	21624.000	21640.602	21657.205	21673.809	21690.415	21707.022	21723.630	21740.240	21756.851	148
149	21773.464	21790.077	21806.692	21823.309	21839.927	21856.546	21873.166	21889.788	21906.411	21923.035	149
150	21939.661	21956.288	21972.917	21989.546	22006.178	22022.810	22039.444	22056.079	22072.715	22089.353	150
151	22105.992	22122.632	22139.274	22155.917	22172.562	22189.207	22205.854	22222.503	22239.152	22255.803	151
152	22272.455	22289.109	22305.764	22322.420	22339.078	22355.737	22372.397	22389.058	22405.721	22422.385	152
153	22439.050	22455.717	22472.385	22489.054	22505.725	22522.397	22539.070	22555.745	22572.421	22589.098	153
154	22605.776	22622.456	22639.137	22655.819	22672.503	22689.188	22705.874	22722.561	22739.250	22755.940	154
155	22772.632	22789.324	22806.018	22822.714	22839.410	22856.108	22872.807	22889.507	22906.209	22922.912	155
156	22939.616	22956.322	22973.029	22989.737	23006.446	23023.157	23039.869	23056.582	23073.296	23090.012	156
157	23106.729	23123.447	23140.167	23156.888	23173.610	23190.333	23207.058	23223.784	23240.511	23257.239	157
158	23273.969	23290.700	23307.432	23324.166	23340.901	23357.637	23374.374	23391.113	23407.853	23424.594	158
159	23441.336	23458.080	23474.824	23491.571	23508.318	23525.067	23541.816	23558.568	23575.320	23592.074	159
160	23608.829	23625.585	23642.342	23659.101	23675.861	23692.622	23709.384	23726.148	23742.913	23759.679	160
161	23776.446	23793.215	23809.984	23826.756	23843.528	23860.301	23877.076	23893.852	23910.630	23927.408	161
162	23944.188	23960.969	23977.751	23994.534	24011.319	24028.105	24044.892	24061.681	24078.470	24095.261	162
163	24112.053	24128.846	24145.641	24162.437	24179.234	24196.032	24212.831	24229.632	24246.434	24263.237	163
164	24280.041	24296.846	24313.653	24330.461	24347.270	24364.C81	24380.892	24397.705	24414.519	24431.334	164
165	24448.151	24464.969	24481.787	24498.607	24515.429	24532.251	24549.075	24565.900	24582.726	24599.553	165
166	24616.382	24633.212	24650.043	24666.875	24683.708	24700.543	24717.379	24734.215	24751.054	24767.893	166
167	24784.734	24801.575	24818.418	24835.262	24852.108	24868.954	24885.802	24902.651	24919.501	24936.352	167
168	24953.205	24970.059	24986.913	25003.770	25020.627	25037.485	25054.345	25071.206	25088.068	25104.931	168
169	25121.795	25138.661	25155.528	25172.395	25189.265	25206.135	25223.006	25239.879	25256.753	25273.628	169
170	25290.504	25307.381	25324.260	25341.140	25358.020	25374.903	25391.786	25408.670	25425.556	25442.442	170
171	25459.330	25476.219	25493.110	25510.001	25526.894	25543.787	25560.682	25577.578	25594.476	25611.374	171
172	25628.274	25645.174	25662.076	25678.979	25695.884	25712.789	25729.696	25746.603	25763.512	25780.422	172
173	25797.333	25814.246	25831.159	25848.074	25864.990	25881.907	25898.825	25915.744	25932.664	25949.586	173
174	25966.509	25983.432	26000.357	26017.284	26034.211	26051.139	26068.069	26085.000	26101.932	26118.865	174
175	26135.799	26152.734	26169.670	26186.608	26203.547	26220.487	26237.428	26254.370	26271.313	26288.258	175
176	26305.203	26322.150	26339.098	26356.047	26372.997	26389.948	26406.900	26423.854	26440.809	26457.764	176
177	26474.721	26491.679	26508.638	26525.599	26542.560	26559.523	26576.486	26593.451	26610.417	26627.384	177
178	26644.352	26661.322	26678.292	26695.264	26712.236	26729.210	26746.185	26763.161	26780.138	26797.116	178
179	26814.096	26831.076	26848.058	26865.041	26882.025	26899.009	26915.996	26932.983	26949.971	26966.960	179
180	26983.951	27000.943	27017.935	27034.929	27051.924	27068.920	27085.917	27102.916	27119.915	27136.916	180
181	27153.917	27170.920	27187.924	27204.929	27221.935	27238.942	27255.950	27272.959	27289.970	27306.981	181
182	27323.994	27341.008	27358.023	27375.039	27392.056	27409.074	27426.093	27443.113	27460.135	27477.157	182
183	27494.181	27511.205	27528.231	27545.258	27562.286	27579.315	27596.345	27613.376	27630.409	27647.442	183
184	27664.477	27681.512	27698.549	27715.587	27732.626	27749.665	27766.707	27783.749	27800.792	27817.836	184
185	27834.881	27851.928	27868.975	27886.024	27903.074	27920.124	27937.176	27954.229	27971.283	27988.338	185
186	28005.394	28022.451	28039.510	28056.569	28073.629	28090.691	28107.753	28124.817	28141.882	28158.948	186
187	28176.014	28193.082	28210.151	28227.221	28244.293	28261.365	28278.438	28295.512	28312.588	28329.664	187
188	28346.742	28363.820	28380.900	28397.981	28415.062	28432.145	28449.229	28466.314	28483.400	28500.487	188
189	28517.575	28534.665	28551.755	28568.846	28585.938	28603.032	28620.126	28637.222	28654.318	28671.416	189
190	28688.515	28705.615	28722.715	28739.817	28756.920	28774.024	28791.129	28808.235	28825.342	28842.450	190
191	28859.560	28876.670	28893.781	28910.893	28928.007	28945.121	28962.237	28979.353	28996.471	29013.589	191
192	29030.709	29047.830	29064.951	29082.074	29099.198	29116.323	29133.449	29150.576	29167.704	29184.833	192
193	29201.963	29219.094	29266.226	29253.359	29270.493	29287.628	29304.765	29321.902	29339.040	29356.179	193
194	29373.320	29390.461	29407.604	29424.747	29441.892	29459.037	29476.184	29493.331	29510.480	29527.629	194
195	29544.780	29561.932	29579.085	29596.238	29613.393	29630.549	29647.706	29664.863	29682.022	29699.182	195
196	29716.343	29733.505	29750.668	29767.832	29784.997	29802.163	29819.330	29836.498	29853.667	29870.837	196
197	29888.008	29905.180	29922.353	29939.527	29956.702	29973.879	29991.056	30008.234	30025.413	30042.593	197
198	30059.774	30076.957	30094.140	30111.324	30128.509	30145.696	30162.883	30180.071	30197.260	30214.451	198
199	30231.642	30248.834	30266.027	30283.222	30300.417	30317.613	30334.811	30352.009	30369.208	30386.409	199
200	30403.610	30420.812	30438.015	30455.220	30472.425	30489.631	30506.839	30524.047	30541.256	30558.467	200
201	30575.678	30592.890	30610.103	30627.318	30644.533	30661.749	30678.966	30696.185	30713.404	30730.624	201
202	30747.845	30765.067	30782.291	30799.515	30816.740	30833.966	30851.193	30868.421	30885.651	30902.881	202
203	30920.112	30937.344	30954.577	30971.811	30989.046	31006.282	31023.519	31040.757	31057.996	31075.236	203
204	31092.477	31109.719	31126.962	31144.205	31161.450	31178.696	31195.943	31213.191	31230.439	31247.689	204
205	31264.940	31282.191	31299.444	31316.698	31333.952	31351.208	31368.464	31385.722	31402.980	31420.240	205
206	31437.500	31454.762	31472.024	31489.288	31506.552	31523.817	31541.083	31558.351	31575.619	31592.888	206
207	31610.158	31627.429	31644.701	31661.974	31679.248	31696.523	31713.799	31731.076	31748.354	31765.633	207
208	31782.912	31800.193	31817.475	31834.757	31852.041	31869.326	31886.611	31903.898	31921.185	31938.473	208
209	31955.763	31973.053	31990.344	32007.637	32024.930	32042.224	32059.519	32076.815	32094.112	32111.410	209
210	32128.709	32146.009	32163.310	32180.611	32197.914	32215.281	32232.522	32249.828	32267.135	32284.442	210

	0	1	2	3	4	5	6	7	8	9	
211	32301. 750	32319. 060	32336. 370	32353. 681	32370. 994	32388. 307	32405. 621	32422. 936	32440. 252	32457. 569	211
212	32474. 887	32492. 205	32509. 525	32526. 846	32544. 167	32561. 490	32578. 814	32596. 138	32613. 463	32630. 790	212
213	32648. 117	32665. 445	32682. 774	32700. 105	32717. 436	32734. 768	32752. 101	32769. 434	32786. 769	32804. 105	213
214	32821. 442	32838. 779	32856. 118	32873. 457	32890. 797	32908. 139	32925. 481	32942. 824	32960. 168	32977. 513	214
215	32994. 859	33012. 206	33029. 554	33046. 903	33064. 253	33081. 603	33098. 955	33116. 307	33133. 661	33151. 015	215
216	33168. 370	33185. 726	33203. 084	33220. 442	33237. 801	33255. 161	33272. 521	33289. 883	33307. 246	33324. 609	216
217	33341. 974	33359. 339	33376. 706	33394. 073	33411. 441	33428. 810	33446. 180	33463. 551	33480. 923	33498. 296	217
218	33515. 669	33533. 044	33550. 420	33567. 796	33585. 173	33602. 552	33619. 931	33637. 311	33654. 692	33672. 074	218
219	33689. 457	33706. 841	33724. 225	33741. 611	33758. 997	33776. 385	33793. 773	33811. 162	33828. 553	33845. 944	219
220	33863. 336	33880. 728	33898. 122	33915. 517	33932. 913	33950. 309	33967. 706	33985. 105	34002. 504	34019. 904	220
221	34037. 305	34054. 707	34072. 110	34089. 514	34106. 918	34124. 324	34141. 730	34159. 138	34176. 546	34193. 955	221
222	34211. 365	34228. 776	34246. 188	34263. 601	34281. 015	34298. 429	34315. 845	34333. 261	34350. 678	34368. 097	222
223	34385. 516	34402. 936	34420. 356	34437. 778	34455. 201	34472. 624	34490. 049	34507. 474	34524. 900	34542. 327	232
224	34559. 756	34577. 184	34594. 614	34612. 045	34629. 477	34646. 909	34664. 342	34681. 777	34699. 212	34716. 648	224
225	34734. 085	34751. 523	34768. 961	34786. 401	34803. 841	34821. 283	34838. 725	34856. 168	34873. 612	34891. 057	225
226	34908. 503	34925. 949	34943. 397	34960. 846	34978. 295	34995. 745	35013. 196	35030. 648	35048. 101	35065. 555	226
227	35083. 009	35100. 465	35117. 921	35135. 379	35152. 837	35170. 296	35187. 756	35205. 216	35222. 678	35240. 141	227
228	35257. 604	35275. 068	35292. 534	35310. 000	35327. 467	35344. 934	35362. 403	35379. 873	35397. 343	35414. 814	228
229	35432. 286	35449. 760	35467. 233	35484. 708	35502. 184	35519. 660	35537. 138	35554. 616	35572. 095	35589. 575	229
230	35607. 056	35624. 538	35642. 021	35659. 504	35676. 989	35694. 474	35711. 960	35729. 447	35746. 935	35764. 423	230
231	35781. 913	35799. 403	35816. 895	35834. 387	35851. 880	35869. 374	35886. 869	35904. 364	35921. 861	35939. 358	231
232	35956. 856	35974. 355	35991. 855	36009. 356	36026. 858	36044. 360	36061. 864	36079. 368	36096. 873	36114. 379	232
233	36131. 886	36149. 393	36166. 902	36184. 411	36201. 922	36219. 433	36236. 945	36254. 457	36271. 971	36289. 486	233
234	36307. 001	36324. 517	36342. 034	36359. 552	36377. 071	36394. 591	36412. 111	36429. 633	36447. 155	36464. 678	234
235	36482. 202	36499. 727	36517. 252	36534. 779	36552. 306	36569. 834	36587. 363	36604. 893	36622. 424	36639. 955	235
236	36657. 488	36675. 021	36692. 555	36710. 090	36727. 626	36745. 162	36762. 700	36780. 238	36797. 778	36815. 318	236
237	36832. 858	36850. 400	36867. 943	36885. 486	36903. 030	36920. 575	36938. 121	36955. 668	36973. 216	36990. 764	237
238	37008. 313	37025. 864	37043. 415	37060. 966	37078. 519	37096. 072	37113. 627	37131. 182	37148. 738	37166. 295	238
239	37183. 853	37201. 411	37218. 970	37236. 531	37254. 092	37271. 653	37289. 216	37306. 780	37324. 344	37341. 909	293
240	37359. 475	37377. 042	37394. 610	37412. 178	37429. 748	37447. 318	37464. 889	37482. 461	37500. 033	37517. 607	240
241	37535. 181	37552. 757	37570. 333	37587. 909	37605. 487	37623. 066	37640. 645	37658. 225	37675. 806	37693. 388	241
242	37710. 970	37728. 554	37746. 138	37763. 723	37781. 309	37798. 896	37816. 484	37834. 072	37851. 661	37869. 251	242
243	37886. 842	37904. 434	37922. 026	37939. 620	37957. 214	37974. 809	37992. 405	38010. 001	38027. 599	38045. 197	243
244	38062. 796	38080. 396	38097. 997	38115. 598	38133. 201	38150. 804	38168. 408	38186. 013	38203. 618	38221. 225	244
245	38238. 832	38256. 440	38274. 049	38291. 659	38309. 269	38326. 881	38344. 493	38362. 106	38379. 720	38397. 334	245
246	38414. 950	38432. 566	38450. 183	38467. 801	38485. 420	38503. 039	38520. 659	38538. 281	38555. 902	38573. 525	246
247	38591. 149	38608. 773	38626. 398	38644. 024	38661. 651	38679. 279	38696. 907	38714. 536	38732. 166	38749. 797	247
248	38767. 429	38785. 061	38802. 694	38820. 328	38837. 963	38855. 599	38873. 235	38890. 873	38908. 511	38926. 149	248
249	38943. 789	38961. 430	38979. 071	38996. 713	39014. 356	39032. 000	39049. 644	39067. 289	39084. 935	39102. 582	249
250	39120. 230	39137. 879	39155. 528	39173. 178	39190. 829	39208. 480	39226. 133	39243. 786	39261. 440	39279. 095	250
251	39296. 751	39314. 407	39332. 065	39349. 723	39367. 382	39385. 041	39402. 702	39420. 363	39438. 025	39455. 688	251
252	39473. 351	39491. 016	39508. 681	39526. 347	39544. 014	39561. 682	39579. 350	39597. 019	39614. 689	39632. 360	252
253	39650. 031	39667. 704	39685. 377	39703. 051	39720. 725	39738. 401	39756. 077	39773. 754	39791. 432	39809. 111	253
254	39826. 790	39844. 471	39862. 152	39879. 833	39897. 516	39915. 199	39932. 884	39950. 568	39968. 254	39985. 941	254
255	40003. 628	40021. 316	40039. 005	40056. 695	40074. 385	40092. 076	40109. 768	40127. 461	40145. 155	40162. 849	255
256	40180. 544	40198. 240	40215. 937	40233. 634	40251. 333	40269. 032	40286. 731	40304. 432	40322. 133	40339. 836	256
257	40357. 538	40375. 242	40392. 947	40410. 652	40428. 358	40446. 065	40463. 772	40481. 481	40499. 190	40516. 900	257
258	40534. 611	40552. 322	40570. 034	40587. 747	40605. 461	40623. 176	40640. 891	40658. 607	40676. 324	40694. 042	258
259	40711. 760	40729. 479	40747. 199	40764. 920	40782. 642	40800. 364	40818. 087	40835. 811	40853. 535	40871. 261	259
260	40888. 987	40906. 714	40924. 442	40942. 170	40959. 899	40977. 629	40995. 360	41013. 092	41030. 824	41048. 557	260
261	41066. 291	41084. 025	41101. 761	41119. 497	41137. 234	41154. 971	41172. 710	41190. 449	41208. 189	41225. 930	261
262	41243. 671	41261. 413	41279. 156	41296. 900	41314. 645	41332. 390	41350. 136	41367. 883	41385. 630	41403. 379	262
263	41421. 128	41438. 878	41456. 628	41474. 380	41492. 132	41509. 885	41527. 638	41545. 393	41563. 148	41580. 904	263
264	41598. 661	41616. 418	41634. 176	41651. 935	41669. 695	41687. 455	41705. 217	41722. 979	41740. 741	41758. 505	264
265	41776. 269	41794. 034	41811. 800	41829. 566	41847. 334	41865. 102	41882. 870	41900. 640	41918. 410	41936. 181	265
266	41953. 953	41971. 726	41989. 499	42007. 273	42025. 048	42042. 823	42060. 600	42078. 377	42096. 154	42113. 933	266
267	42131. 712	42149. 492	42167. 273	42185. 055	42202. 837	42220. 620	42238. 404	42256. 188	42273. 973	42291. 760	267
268	42309. 546	42327. 334	42345. 122	42362. 911	42380. 701	42398. 491	42416. 283	42434. 075	42451. 867	42469. 661	268
269	42487. 455	42505. 250	42523. 046	42540. 842	42558. 639	42576. 437	42594. 236	42612. 035	42629. 836	42647. 636	269
270	42665. 438	42683. 240	42701. 044	42718. 847	42736. 652	42754. 457	42772. 263	42790. 070	42807. 878	42825. 686	270
271	42843. 495	42861. 305	42879. 115	42896. 927	42914. 739	42932. 551	42950. 365	42968. 179	42985. 994	43003. 810	271
272	43021. 626	43039. 443	43057. 261	43075. 080	43092. 899	43110. 719	43128. 540	43146. 362	43164. 184	43182. 007	272
273	43199. 831	43217. 655	43235. 480	43253. 306	43271. 133	43288. 960	43306. 788	43324. 617	43342. 447	43360. 277	273
274	43378. 108	43395. 940	43413. 773	43431. 606	43449. 440	43467. 275	43485. 110	43502. 946	43520. 783	43538. 621	274
275	43556. 459	43574. 298	43592. 138	43609. 978	43627. 820	43645. 662	43663. 504	43681. 348	43699. 192	43717. 037	275
276	43734. 882	43752. 729	43770. 576	43788. 424	43806. 272	43824. 121	43841. 971	43859. 822	43877. 673	43895. 526	276
277	43913. 378	43931. 232	43949. 086	43966. 941	43984. 797	44002. 653	44020. 511	44038. 368	44056. 227	44074. 086	277
278	44091. 946	44109. 807	44127. 669	44145. 531	44163. 394	44181. 258	44199. 122	44216. 987	44234. 853	44252. 719	278
279	44270. 587	44288. 454	44306. 323	44324. 193	44342. 063	44359. 933	44377. 805	44395. 677	44413. 550	44431. 424	279
280	44449. 298	44467. 173	44485. 049	44502. 926	44520. 803	44538. 681	44556. 560	44574. 439	44592. 319	44610. 200	280

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281	44628. 081	44645. 964	44663. 847	44681. 730	44699. 615	44717. 500	44735. 386	44753. 272	44771. 159	44789. 047	281
282	44806. 936	44824. 825	44842. 715	44860. 606	44878. 497	44896. 390	44914. 282	44932. 176	44950. 070	44967. 965	282
283	44985. 861	45003. 757	45021. 655	45039. 552	45057. 451	45075. 350	45093. 250	45111. 151	45129. 052	45146. 954	283
284	45164. 857	45182. 760	45200. 665	45218. 569	45236. 475	45254. 381	45272. 288	45290. 196	45308. 104	45326. 014	284
285	45343. 923	45361. 834	45379. 745	45397. 657	45415. 570	45433. 483	45451. 397	45469. 312	45487. 227	45505. 143	285
286	45523. 060	45540. 977	45558. 896	45576. 814	45594. 734	45612. 654	45630. 575	45648. 497	45666. 419	45684. 343	286
287	45702. 266	45720. 191	45738. 116	45756. 042	45773. 968	45791. 896	45809. 824	45827. 752	45845. 682	45863. 612	287
288	45881. 542	45899. 474	45917. 406	45935. 339	45953. 272	45971. 207	45989. 142	46007. 077	46025. 013	46042. 950	288
289	46060. 888	46078. 826	46096. 765	46114. 705	46132. 646	46150. 587	46168. 529	46186. 471	46204. 414	46222. 358	289
290	46240. 303	46258. 248	46276. 194	46294. 141	46312. 088	46330. 036	46347. 985	46365. 934	46383. 884	46401. 835	290
291	46419. 787	46437. 739	46455. 692	46473. 645	46491. 599	46509. 554	46527. 510	46545. 466	46563. 423	46581. 381	291
292	46599. 339	46617. 298	46635. 258	46653. 218	46671. 179	46689. 141	46707. 104	46725. 067	46743. 030	46760. 995	292
293	46778. 960	46796. 926	46814. 893	46832. 860	46850. 828	46868. 796	46886. 765	46904. 735	46922. 706	46940. 677	293
294	46958. 649	46976. 622	46994. 595	47012. 569	47030. 544	47048. 519	47066. 496	47084. 472	47102. 450	47120. 428	294
295	47138. 407	47156. 386	47174. 366	47192. 347	47210. 329	47228. 311	47246. 294	47264. 277	47282. 261	47300. 246	295
296	47318. 232	47336. 218	47354. 205	47372. 192	47390. 181	47408. 170	47426. 159	47444. 149	47462. 140	47480. 132	296
297	47498. 124	47516. 117	47534. 111	47552. 105	47570. 100	47588. 096	47606. 092	47624. 089	47642. 087	47660. 085	297
298	47678. 084	47696. 084	47714. 084	47732. 085	47750. 087	47768. 089	47786. 093	47804. 096	47822. 101	47840. 106	298
299	47858. 111	47876. 118	47894. 125	47912. 133	47930. 141	47948. 150	47966. 160	47984. 170	48002. 181	48020. 193	299
300	48038. 205	48056. 218	48074. 232	48092. 247	48110. 262	48128. 277	48146. 294	48164. 311	48182. 329	48200. 347	300
301	48218. 366	48236. 386	48254. 406	48272. 427	48290. 449	48308. 471	48326. 494	48344. 518	48362. 542	48380. 567	301
302	48398. 593	48416. 619	48434. 646	48452. 674	48470. 703	48488. 732	48506. 761	48524. 792	48542. 822	48560. 854	302
303	48578. 886	48596. 919	48614. 953	48632. 987	48651. 022	48669. 058	48687. 094	48705. 131	48723. 169	48741. 207	303
304	48759. 246	48777. 285	48795. 326	48813. 366	48831. 408	48849. 450	48867. 493	48885. 536	48903. 581	48921. 625	304
305	48939. 671	48957. 717	48975. 764	48993. 811	49011. 859	49029. 908	49047. 957	49066. 007	49084. 058	49102. 110	305
306	49120. 162	49138. 214	49156. 268	49174. 322	49192. 376	49210. 431	49228. 487	49246. 544	49264. 601	49282. 659	306
307	49300. 718	49318. 777	49336. 837	49354. 897	49372. 958	49391. 020	49409. 083	49427. 146	49445. 209	49463. 274	307
308	49481. 339	49499. 404	49517. 471	49535. 538	49553. 605	49571. 674	49589. 743	49607. 812	49625. 883	49643. 953	308
309	49662. 025	49680. 097	49698. 170	49716. 243	49734. 318	49752. 392	49770. 468	49788. 544	49806. 620	49824. 698	309
310	49842. 776	49860. 854	49878. 934	49897. 014	49915. 094	49933. 175	49951. 257	49969. 340	49987. 423	50005. 507	310
311	50023. 591	50041. 676	50059. 762	50077. 848	50095. 935	50114. 023	50132. 111	50150. 200	50168. 290	50186. 380	311
312	50204. 471	50222. 562	50240. 654	50258. 747	50276. 841	50294. 935	50313. 029	50331. 125	50349. 221	50367. 317	312
313	50385. 415	50403. 513	50421. 611	50439. 710	50457. 810	50475. 910	50494. 012	50512. 113	50530. 216	50548. 319	313
314	50566. 422	50584. 527	50602. 631	50620. 737	50638. 843	50656. 950	50675. 058	50693. 166	50711. 274	50729. 384	314
315	50747. 494	50765. 604	50783. 716	50801. 827	50819. 940	50838. 053	50856. 167	50874. 281	50892. 397	50910. 512	315
316	50928. 629	50946. 746	50964. 863	50982. 981	51001. 100	51019. 220	51037. 340	51055. 461	51073. 582	51091. 704	316
317	51109. 827	51127. 950	51146. 074	51164. 198	51182. 324	51200. 449	51218. 576	51236. 703	51254. 831	51272. 959	317
318	51291. 088	51309. 218	51327. 348	51345. 479	51363. 610	51381. 742	51399. 875	51418. 008	51436. 142	51454. 277	318
319	51472. 412	51490. 548	51508. 684	51526. 822	51544. 959	51563. 098	51581. 237	51599. 376	51617. 517	51635. 657	319
320	51653. 799	51671. 941	51690. 084	51708. 227	51726. 371	51744. 516	51762. 661	51780. 807	51798. 953	51817. 101	320
321	51835. 248	51853. 397	51871. 546	51889. 695	51907. 845	51925. 996	51944. 148	51962. 300	51980. 453	51998. 606	321
322	52016. 760	52034. 915	52053. 070	52071. 226	52089. 382	52107. 539	52125. 697	52143. 855	52162. 014	52180. 174	322
323	52198. 334	52216. 494	52234. 656	52252. 818	52270. 981	52289. 144	52307. 308	52325. 472	52343. 637	52361. 803	323
324	52379. 969	52398. 136	52416. 304	52434. 472	52452. 641	52470. 810	52488. 980	52507. 151	52525. 322	52543. 494	324
325	52561. 667	52579. 840	52598. 014	52616. 188	52634. 363	52652. 539	52670. 715	52688. 892	52707. 069	52725. 247	325
326	52743. 426	52761. 605	52779. 785	52797. 965	52816. 146	52834. 328	52852. 511	52870. 693	52888. 877	52907. 061	326
327	52925. 246	52943. 431	52961. 617	52979. 804	52997. 991	53016. 179	53034. 368	53052. 557	53070. 746	53088. 937	327
328	53107. 127	53125. 319	53143. 511	53161. 704	53179. 897	53198. 091	53216. 286	53234. 481	53252. 677	53270. 873	328
329	53289. 070	53307. 267	53325. 466	53343. 664	53361. 864	53380. 064	53398. 265	53416. 466	53434. 668	53452. 870	329
330	53471. 073	53489. 277	53507. 481	53525. 686	53543. 891	53562. 097	53580. 304	53598. 511	53616. 719	53634. 928	330
331	53653. 137	53671. 347	53689. 557	53707. 768	53725. 979	53744. 192	53762. 404	53780. 618	53798. 832	53817. 046	331
332	53835. 261	53853. 477	53871. 693	53889. 910	53908. 128	53926. 346	53944. 565	53962. 784	53981. 004	53999. 225	332
333	54017. 446	54035. 667	54053. 890	54072. 113	54090. 336	54108. 560	54126. 785	54145. 011	54163. 237	54181. 463	333
334	54199. 690	54217. 918	54236. 146	54254. 375	54272. 605	54290. 835	54309. 066	54327. 297	54345. 529	54363. 762	334
335	54381. 995	54400. 228	54418. 463	54436. 698	54454. 933	54473. 169	54491. 406	54509. 643	54527. 881	54546. 120	335
336	54564. 359	54582. 598	54600. 839	54619. 080	54637. 321	54655. 563	54673. 806	54692. 049	54710. 293	54728. 537	336
337	54746. 782	54765. 028	54783. 274	54801. 521	54819. 769	54838. 017	54856. 265	54874. 514	54892. 764	54911. 014	337
338	54929. 265	54947. 517	54965. 769	54984. 022	55002. 275	55020. 529	55038. 784	55057. 039	55075. 294	55093. 551	338
339	55111. 808	55130. 065	55148. 323	55166. 582	55184. 841	55203. 101	55221. 361	55239. 622	55257. 884	55276. 146	339
340	55294. 409	55312. 672	55330. 936	55349. 201	55367. 466	55385. 731	55403. 998	55422. 265	55440. 532	55458. 800	340
341	55477. 069	55495. 338	55513. 608	55531. 878	55550. 149	55568. 421	55586. 693	55604. 966	55623. 239	55641. 513	341
342	55659. 787	55678. 063	55696. 338	55714. 614	55732. 891	55751. 169	55769. 447	55787. 725	55806. 004	55824. 284	342
343	55842. 565	55860. 846	55879. 127	55897. 409	55915. 692	55933. 975	55952. 259	55970. 543	55988. 828	56007. 114	343
344	56025. 400	56043. 687	56061. 974	56080. 262	56098. 550	56116. 840	56135. 129	56153. 419	56171. 710	56190. 002	344
345	56208. 294	56226. 586	56244. 879	56263. 173	56281. 467	56299. 762	56318. 058	56336. 354	56354. 650	56372. 947	345
346	56391. 245	56409. 543	56427. 842	56446. 142	56464. 442	56482. 743	56501. 044	56519. 346	56537. 648	56555. 951	346
347	56574. 254	56592. 559	56610. 863	56629. 169	56647. 474	56665. 781	56684. 088	56702. 395	56720. 703	56739. 012	347
348	56757. 321	56775. 631	56793. 942	56812. 253	56830. 564	56848. 877	56867. 189	56885. 503	56903. 816	56922. 131	348
349	56940. 446	56958. 762	56977. 078	56995. 394	57013. 712	57032. 030	57050. 348	57068. 667	57086. 987	57105. 307	349
350	57123. 628	57141. 949	57160. 271	57178. 593	57196. 916	57215. 240	57233. 564	57251. 889	57270. 214	57288. 540	350

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Table of 2n ln n *for values of* n *from 1 to 10,000*—Continued

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351	57306.867	57325.194	57343.521	57361.849	57380.178	57398.507	57416.837	57435.168	57453.499	57471.830	351
352	57490.163	57508.495	57526.829	57545.162	57563.497	57581.832	57600.167	57618.503	57636.840	57655.177	352
353	57673.515	57691.854	57710.193	57728.532	57746.872	57765.213	57783.554	57801.896	57820.238	57838.581	353
354	57856.925	57875.269	57893.613	57911.958	57930.304	57948.650	57966.997	57985.345	58003.693	58022.041	354
355	58040.390	58058.740	58077.090	58095.441	58113.793	58132.144	58150.497	58168.850	58187.204	58205.558	355
356	58223.913	58242.268	58260.624	58278.980	58297.337	58315.695	58334.053	58352.412	58370.771	58389.131	356
357	58407.491	58425.852	58444.213	58462.575	58480.938	58499.301	58517.665	58536.029	58554.394	58572.759	357
358	58591.125	58609.492	58627.859	58646.227	58664.595	58682.964	58701.333	58719.703	58738.073	58756.444	358
359	58774.816	58793.188	58811.560	58829.934	58848.307	58866.682	58885.057	58903.432	58921.808	58940.185	359
360	58958.562	58976.939	58995.318	59013.696	59032.076	59050.455	59068.836	59087.217	59105.598	59123.981	360
361	59142.363	59160.746	59179.130	59197.514	59215.899	59234.285	59252.671	59271.057	59289.444	59307.832	361
362	59326.220	59344.609	59362.998	59381.388	59399.778	59418.169	59436.561	59454.953	59473.345	59491.739	362
363	59510.132	59528.527	59546.921	59565.317	59583.713	59602.109	59620.506	59638.904	59657.302	59675.700	363
364	59694.100	59712.499	59730.900	59749.300	59767.702	59786.104	59804.506	59822.909	59841.313	59859.717	364
365	59878.122	59896.527	59914.933	59933.339	59951.746	59970.153	59988.561	60006.970	60025.379	60043.789	365
366	60062.199	60080.610	60099.021	60117.433	60135.845	60154.258	60172.671	60191.085	60209.500	60227.915	366
367	60246.330	60264.747	60283.163	60301.581	60319.998	60338.417	60356.836	60375.255	60393.675	60412.096	367
368	60430.517	60448.938	60467.360	60485.783	60504.206	60522.630	60541.054	60559.479	60577.905	60596.331	368
369	60614.757	60633.184	60651.612	60670.040	60688.469	60706.898	60725.328	60743.758	60762.189	60780.620	369
370	60799.052	60817.484	60835.917	60854.351	60872.785	60891.220	60909.655	60928.090	60946.527	60964.963	370
371	60983.401	61001.838	61020.277	61038.716	61057.155	61075.595	61094.036	61112.477	61130.918	61149.361	371
372	61167.803	61186.247	61204.690	61223.135	61241.579	61260.025	61278.471	61296.917	61315.364	61333.812	372
373	61352.260	61370.708	61389.157	61407.607	61426.057	61444.508	61462.959	61481.411	61499.864	61518.316	373
374	61536.770	61555.224	61573.678	61592.133	61610.589	61629.045	61647.502	61665.959	61684.416	61702.875	374
375	61721.333	61739.793	61758.252	61776.713	61795.174	61813.635	61832.097	61850.560	61869.023	61887.486	375
376	61905.950	61924.415	61942.880	61961.346	61979.812	61998.279	62016.746	62035.214	62053.682	62072.151	376
377	62090.620	62109.090	62127.561	62146.032	62164.503	62182.975	62201.448	62219.921	62238.395	62256.869	377
378	62275.343	62293.819	62312.294	62330.771	62349.247	62367.725	62386.203	62404.681	62423.160	62441.639	378
379	62460.119	62478.600	62497.081	62515.562	62534.045	62552.527	62571.010	62589.494	62607.978	62626.463	379
380	62644.948	62663.434	62681.920	62700.407	62718.894	62737.382	62755.871	62774.360	62792.849	62811.339	380
381	62829.830	62848.321	62866.812	62885.304	62903.797	62922.290	62940.784	62959.278	62977.773	62996.268	381
382	63014.764	63023.260	63051.757	63070.254	63088.752	63107.250	63125.749	63144.248	63162.748	63181.249	382
383	63199.750	63218.251	63236.753	63255.256	63273.759	63292.263	63310.767	63329.271	63347.776	63366.282	383
384	63384.788	63403.295	63421.802	63440.310	63458.818	63477.327	63495.836	63514.346	63532.857	63551.367	384
385	63569.879	63588.391	63606.903	63625.416	63643.930	63662.444	63680.958	63699.473	63717.989	63736.505	385
386	63755.021	63773.538	63792.056	63810.574	63829.093	63847.612	63866.132	63884.652	63903.173	63921.694	386
387	63940.216	63958.738	63977.261	63995.784	64014.308	64032.832	64051.357	64069.883	64088.408	64106.935	387
388	64125.462	64143.989	64162.517	64181.046	64199.575	64218.104	64236.634	64255.165	64273.696	64292.227	388
389	64310.759	64329.292	64347.825	64366.359	64384.893	64403.427	64421.963	64440.498	64459.034	64477.571	389
390	64496.108	64514.646	64533.184	64551.723	64570.262	64588.802	64607.342	64625.883	64644.424	64662.966	390
391	64681.509	64700.051	64718.595	64737.139	64755.683	64774.228	64792.773	64811.319	64829.866	64848.412	391
392	64866.960	64885.508	64904.056	64922.605	64941.155	64959.705	64978.255	64996.806	65015.358	65033.910	392
393	65052.462	65071.015	65089.569	65108.123	65126.678	65145.233	65163.788	65182.344	65200.901	65219.458	393
394	65238.016	65256.574	65275.132	65293.692	65312.251	65330.811	65349.372	65367.933	65386.495	65405.057	394
395	65423.620	65442.183	65460.747	65479.311	65497.876	65516.441	65535.006	65553.573	65572.139	65590.707	395
396	65609.274	65627.843	65646.411	65664.981	65683.550	65702.121	65720.692	65739.263	65757.835	65776.407	396
397	65794.980	65813.553	65832.127	65850.701	65869.276	65887.851	65906.427	65925.003	65943.580	65962.157	397
398	65980.735	65999.314	66017.892	66036.472	66055.052	66073.632	66092.213	66110.794	66129.376	66147.958	398
399	66166.541	66185.124	66203.708	66222.293	66240.877	66259.463	66278.049	66296.635	66315.222	66333.809	399
400	66352.397	66370.985	66389.574	66408.164	66426.753	66445.344	66463.935	66482.526	66501.118	66519.710	400
401	66538.303	66556.896	66575.490	66594.085	66612.679	66631.275	66649.871	66668.467	66687.064	66705.661	401
402	66724.259	66742.857	66761.456	66780.055	66798.655	66817.255	66835.856	66854.458	66873.059	66891.662	402
403	66910.264	66928.868	66947.472	66966.076	66984.681	67003.286	67021.892	67040.498	67059.105	67077.712	403
404	67096.320	67114.928	67133.537	67152.146	67170.756	67189.366	67207.977	67226.588	67245.200	67263.812	404
405	67282.424	67301.038	67319.651	67338.265	67356.880	67375.495	67394.111	67412.727	67431.344	67449.961	405
406	67468.579	67487.197	67505.815	67524.434	67543.054	67561.674	67580.295	67598.916	67617.537	67636.159	406
407	67654.782	67672.405	67692.029	67710.653	67729.277	67747.902	67766.528	67785.154	67803.780	67822.407	407
408	67841.034	67859.662	67878.291	67896.920	67915.549	67934.179	67952.809	67971.440	67990.072	68008.704	408
409	68027.336	68045.969	68064.602	68083.236	68101.870	68120.505	68139.140	68157.776	68176.412	68195.049	409
410	68213.686	68232.324	68250.962	68269.601	68288.240	68306.880	68325.520	68344.161	68362.802	68381.444	410
411	68400.086	68418.728	68437.371	68456.015	68474.659	68493.304	68511.949	68530.594	68549.240	68567.887	411
412	68586.534	68605.181	68623.829	68642.477	68661.126	68679.776	68698.426	68717.076	68735.727	68754.378	412
413	68773.030	68791.682	68810.335	68828.988	68847.642	68866.296	68884.951	68903.606	68922.262	68940.918	413
414	68959.575	68978.232	68996.890	69015.548	69034.206	69052.865	69071.525	69090.185	69108.845	69127.506	414
415	69146.168	69164.830	69183.492	69202.155	69220.819	69239.483	69258.147	69276.812	69295.477	69314.143	415
416	69332.809	69351.476	69370.143	69388.811	69407.479	69426.148	69444.817	69463.487	69482.157	69500.828	416
417	69519.499	69538.170	69556.842	69575.515	69594.188	69612.861	69631.535	69650.210	69668.885	69687.560	417
418	69706.236	69724.912	69743.589	69762.267	69780.944	69799.623	69818.302	69836.981	69855.660	69874.341	418
419	69893.021	69911.702	69930.384	69949.066	69967.749	69986.432	70005.115	70023.799	70042.484	70061.169	419
420	70079.854	70098.540	70117.227	70135.913	70154.601	70173.289	70191.977	70210.666	70229.355	70248.045	420

	0	1	2	3	4	5	6	7	8	9	
421	70266.735	70285.426	70304.117	70322.808	70341.500	70360.193	70378.886	70397.580	70416.274	70434.968	421
422	70453.663	70472.358	70491.054	70509.751	70528.448	70547.145	70565.843	70584.541	70603.240	70621.939	422
423	70640.638	70659.339	70678.039	70696.740	70715.442	70734.144	70752.846	70771.549	70790.253	70808.957	423
424	70827.661	70846.366	70865.071	70883.777	70902.484	70921.190	70939.898	70958.605	70977.313	70996.022	424
425	71014.731	71033.441	71052.151	71070.861	71089.572	71108.284	71126.996	71145.708	71164.421	71183.134	425
426	71201.848	71220.562	71239.277	71257.992	71276.708	71295.424	71314.141	71332.858	71351.576	71370.294	426
427	71389.012	71407.731	71426.451	71445.170	71463.891	71482.612	71501.333	71520.055	71538.777	71557.500	427
428	71576.223	71594.947	71613.671	71632.395	71651.120	71669.846	71688.572	71707.298	71726.025	71744.753	428
429	71763.480	71782.209	71800.938	71819.667	71838.396	71857.127	71875.857	71894.588	71913.320	71932.052	429
430	71950.785	71969.518	71988.251	72006.985	72025.719	72044.454	72063.189	72081.925	72100.661	72119.398	430
431	72138.135	72156.873	72175.611	72194.349	72213.088	72231.828	72250.568	72269.308	72288.049	72306.790	431
432	72325.532	72344.274	72363.017	72381.760	72400.504	72419.248	72437.993	72456.738	72475.483	72494.229	432
433	72512.976	72531.722	72550.470	72569.218	72587.966	72606.715	72625.464	72644.213	72662.964	72681.714	433
434	72700.465	72719.217	72737.969	72756.721	72775.474	72794.227	72812.981	72831.735	72850.490	72869.245	434
435	72888.001	72906.757	72925.513	72944.270	72963.028	72981.786	73000.544	73019.303	73038.062	73056.822	435
436	73075.582	73094.343	73113.104	73131.866	73150.628	73169.390	73188.153	73206.917	73225.681	73244.445	436
437	73263.210	73281.975	73300.741	73319.507	73338.274	73357.041	73375.808	73394.576	73413.345	73432.114	437
438	73450.883	73469.653	73488.423	73507.194	73525.965	73544.737	73563.509	73582.281	73601.054	73619.828	438
439	73638.602	73657.376	73676.151	73694.926	73713.702	73732.478	73751.255	73770.032	73788.810	73807.588	439
440	73826.366	73845.145	73863.925	73882.705	73901.485	73920.266	73939.047	73957.829	73976.611	73995.393	440
441	74014.176	74032.960	74051.744	74070.528	74089.313	74108.098	74126.884	74145.670	74164.457	74183.244	441
442	74202.032	74220.820	74239.608	74258.397	74277.186	74295.976	74314.766	74333.557	74352.348	74371.140	442
443	74389.932	74408.725	74427.518	74446.311	74465.105	74483.899	74502.694	74521.489	74540.285	74559.081	443
444	74577.878	74596.675	74615.472	74634.270	74653.069	74671.867	74690.667	74709.466	74728.267	74747.067	444
445	74765.868	74784.670	74803.472	74822.274	74841.077	74859.881	74878.684	74897.489	74916.293	74935.098	445
446	74953.904	74972.710	74991.517	75010.323	75029.131	75047.939	75066.747	75085.556	75104.365	75123.174	446
447	75141.985	75160.795	75179.606	75198.417	75217.229	75236.042	75254.854	75273.667	75292.481	75311.295	447
448	75330.110	75348.925	75367.740	75386.556	75405.372	75424.189	75443.006	75461.824	75480.642	75499.461	448
449	75518.280	75537.099	75555.919	75574.739	75593.560	75612.381	75631.203	75650.025	75668.848	75687.671	449
450	75706.494	75725.318	75744.142	75762.967	75781.792	75800.618	75819.444	75838.271	75857.098	75875.925	450
451	75894.753	75913.581	75932.410	75951.239	75970.069	75988.899	76007.729	76026.560	76045.392	76064.224	451
452	76083.056	76101.889	76120.722	76139.556	76158.390	76177.224	76196.059	76214.895	76233.731	76252.567	452
453	76271.404	76290.241	76309.078	76327.916	76346.755	76365.594	76384.433	76403.273	76422.113	76440.954	453
454	76459.795	76478.637	76497.479	76516.321	76535.164	76554.007	76572.851	76591.695	76610.540	76629.385	454
455	76648.231	76667.077	76685.923	76704.770	76723.617	76742.465	76761.313	76780.162	76799.011	76817.860	455
456	76836.710	76855.561	76874.412	76893.263	76912.115	76930.967	76949.819	76968.672	76987.526	77006.380	456
457	77025.234	77044.089	77062.944	77081.799	77100.656	77119.512	77138.369	77157.226	77176.084	77194.942	457
458	77213.801	77232.660	77251.520	77270.380	77289.240	77308.101	77326.962	77345.824	77364.686	77383.549	458
459	77402.412	77421.276	77440.139	77459.004	77477.869	77496.734	77515.599	77534.466	77553.332	77572.199	459
460	77591.067	77609.934	77628.803	77647.671	77666.540	77685.410	77704.280	77723.151	77742.021	77760.893	460
461	77779.764	77798.637	77817.509	77836.382	77855.256	77874.130	77893.004	77911.879	77930.754	77949.630	461
462	77968.506	77987.382	78006.259	78025.137	78044.014	78062.893	78081.771	78100.651	78119.530	78138.410	462
463	78157.290	78176.171	78195.053	78213.934	78232.816	78251.699	78270.582	78289.465	78308.349	78327.234	463
464	78346.118	78365.003	78383.889	78402.775	78421.661	78440.548	78459.436	78478.323	78497.212	78516.100	464
465	78534.989	78553.879	78572.769	78591.659	78610.550	78629.441	78648.332	78667.224	78686.117	78705.010	465
466	78723.903	78742.797	78761.691	78780.586	78799.481	78818.376	78837.272	78856.168	78875.065	78893.962	466
467	78912.860	78931.758	78950.657	78969.555	78988.455	79007.354	79026.255	79045.155	79064.056	79082.958	467
468	79101.860	79120.762	79139.665	79158.568	79177.472	79196.376	79215.280	79234.185	79253.090	79271.996	468
469	79290.902	79309.809	79328.716	79347.623	79366.531	79385.439	79404.348	79423.257	79442.167	79461.077	469
470	79479.987	79498.898	79517.809	79536.721	79555.633	79574.546	79593.459	79612.372	79631.286	79650.200	470
471	79669.115	79688.030	79706.945	79725.861	79744.778	79763.695	79782.612	79801.529	79820.447	79839.366	471
472	79858.285	79877.204	79896.124	79915.044	79933.965	79952.886	79971.807	79990.729	80009.651	80028.574	472
473	80047.497	80066.421	80085.345	80104.269	80123.194	80142.119	80161.045	80179.971	80198.898	80217.825	473
474	80236.752	80255.680	80274.608	80293.537	80312.466	80331.395	80350.325	80369.255	80388.186	80407.117	474
475	80426.049	80444.981	80463.913	80482.846	80501.780	80520.713	80539.647	80558.582	80577.517	80596.452	475
476	80615.388	80634.324	80653.261	80672.198	80691.135	80710.073	80729.012	80747.950	80766.889	80785.829	476
477	80804.769	80823.709	80842.650	80861.592	80880.533	80899.475	80918.418	80937.361	80956.304	80975.248	477
478	80994.192	81013.137	81032.082	81051.027	81069.973	81088.919	81107.866	81126.813	81145.761	81164.709	478
479	81183.657	81202.606	81221.555	81240.504	81259.454	81278.405	81297.356	81316.307	81335.259	81354.211	479
480	81373.163	81392.116	81411.070	81430.024	81448.978	81467.932	81486.887	81505.843	81524.799	81543.755	480
481	81562.712	81581.669	81600.626	81619.584	81638.543	81657.501	81676.461	81695.420	81714.380	81733.341	481
482	81752.302	81771.263	81790.224	81809.187	81828.149	81847.112	81866.075	81885.039	81904.003	81922.968	482
483	81941.933	81960.898	81979.864	81998.830	82017.797	82036.764	82055.731	82074.699	82093.668	82112.636	483
484	82131.606	82150.575	82169.545	82188.515	82207.486	82226.457	82245.429	82264.401	82283.373	82302.346	484
485	82321.320	82340.293	82359.267	82378.242	82397.217	82416.192	82435.168	82454.144	82473.121	82492.097	485
486	82511.075	82530.053	82549.031	82568.009	82586.989	82605.968	82624.948	82643.928	82662.909	82681.890	486
487	82700.871	82719.853	82738.836	82757.818	82776.801	82795.785	82814.769	82833.753	82852.738	82871.723	487
488	82890.709	82909.695	82928.681	82947.668	82966.655	82985.643	83004.631	83023.619	83042.608	83061.598	488
489	83080.587	83099.577	83118.568	83137.559	83156.550	83175.542	83194.534	83213.527	83232.520	83251.513	489
490	83270.507	83289.501	83308.495	83327.490	83346.486	83365.482	83384.478	83403.475	83422.472	83441.469	490

Table of 2n ln n *for values of* n *from 1 to 10,000*—Continued

	0	1	2	3	4	5	6	7	8	9	
491	83460.467	83479.465	83498.464	83517.463	83536.462	83555.462	83574.463	83593.463	83612.464	83631.466	491
492	83650.468	83669.470	83688.473	83707.476	83726.480	83745.484	83764.488	83783.493	83802.498	83821.503	492
493	83840.509	83859.516	83878.523	83897.530	83916.537	83935.545	83954.554	83973.563	83992.572	84011.582	493
494	84030.592	84049.602	84068.613	84087.624	84106.636	84125.648	84144.660	84163.673	84182.686	84201.700	494
495	84220.714	84239.729	84258.744	84277.759	84296.775	84315.791	84334.807	84353.824	84372.841	84391.859	495
496	84410.877	84429.896	84448.915	84467.934	84486.954	84505.974	84524.994	84544.015	84563.037	84582.058	496
497	84601.081	84620.103	84639.126	84658.149	84677.173	84696.197	84715.222	84734.247	84753.272	84772.298	497
498	84791.324	84810.351	84829.378	84848.405	84867.433	84886.461	84905.490	84924.519	84943.548	84962.578	498
499	84981.608	85000.639	85019.670	85038.701	85057.733	85076.765	85095.798	85114.831	85133.864	85152.898	499
500	85171.932	85190.966	85210.001	85229.037	85248.073	85267.109	85286.145	85305.182	85324.220	85343.258	500
501	85362.296	85381.334	85400.373	85419.413	85438.452	85457.493	85476.533	85495.574	85514.616	85533.657	501
502	85552.699	85571.742	85590.785	85609.828	85628.872	85647.916	85666.961	85686.006	85705.051	85724.097	502
503	85743.143	85762.190	85781.237	85800.284	85819.332	85838.380	85857.428	85876.477	85895.527	85914.576	503
504	85933.626	85952.677	85971.728	85990.779	86009.831	86028.883	86047.936	86066.988	86086.042	86105.095	504
505	86124.150	86143.204	86162.259	86181.314	86200.370	86219.426	86238.482	86257.539	86276.596	86295.654	505
506	86314.712	86333.771	86352.829	86371.889	86390.948	86410.008	86429.069	86448.130	86467.191	86486.252	506
507	86505.314	86524.377	86543.440	86562.503	86581.566	86600.630	86619.695	86638.759	86657.825	86676.890	507
508	86695.956	86715.002	86734.089	86753.156	86772.224	86791.292	86810.360	86829.429	86848.498	86867.567	508
509	86886.637	86905.707	86924.778	86943.849	86962.920	86981.992	87001.064	87020.137	87039.210	87058.284	509
510	87077.357	87096.431	87115.506	87134.581	87153.656	87172.732	87191.808	87210.885	87229.962	87249.039	510
511	87268.117	87287.195	87306.273	87325.352	87344.432	87363.511	87382.591	87401.672	87420.753	87439.834	511
512	87458.915	87477.997	87497.080	87516.163	87535.246	87554.329	87573.413	87592.498	87611.583	87630.668	512
513	87649.753	87668.839	87687.925	87707.012	87726.099	87745.187	87764.275	87783.363	87802.451	87821.540	513
514	87840.630	87859.720	87878.810	87897.900	87916.991	87936.083	87955.175	87974.267	87993.359	88012.452	514
515	88031.545	88050.639	88069.733	88088.828	88107.923	88127.018	88146.113	88165.210	88184.306	88203.403	515
516	88222.500	88241.598	88260.695	88279.794	88298.893	88317.992	88337.091	88356.191	88375.291	88394.392	516
517	88413.493	88432.595	88451.696	88470.799	88489.901	88509.004	88528.108	88547.211	88566.316	88585.420	517
518	88604.525	88623.630	88642.736	88661.842	88680.949	88700.055	88719.163	88738.270	88757.378	88776.487	518
519	88795.596	88814.705	88833.814	88852.924	88872.035	88891.145	88910.256	88929.368	88948.480	88967.592	519
520	88986.705	89005.818	89024.931	89044.045	89063.159	89082.273	89101.388	89120.504	89139.619	89158.736	520
521	89177.852	89196.969	89216.086	89235.204	89254.322	89273.440	89292.559	89311.678	89330.798	89349.918	521
522	89369.038	89388.159	89407.280	89426.401	89445.523	89464.645	89483.768	89502.891	89522.014	89541.138	522
523	89560.262	89579.387	89598.512	89617.637	89636.763	89655.889	89675.015	89694.142	89713.269	89732.397	523
524	89751.525	89770.653	89789.782	89808.911	89828.040	89847.170	89866.300	89885.431	89904.562	89923.693	524
525	89942.825	89961.957	89981.090	90000.223	90019.356	90038.490	90057.624	90076.758	90095.893	90115.028	525
526	90134.164	90153.300	90172.436	90191.573	90210.710	90229.847	90248.985	90268.124	90287.262	90306.401	526
527	90325.541	90344.680	90363.821	90382.961	90402.102	90421.243	90440.385	90459.527	90478.669	90497.812	527
528	90516.955	90536.099	90555.243	90574.387	90593.532	90612.677	90631.822	90650.968	90670.114	90689.261	528
529	90708.408	90727.555	90746.703	90765.851	90784.999	90804.148	90823.298	90842.447	90861.597	90880.747	529
530	90899.898	90919.049	90938.201	90957.353	90976.505	90995.658	91014.811	91033.964	91053.118	91072.272	530
531	91091.426	91110.581	91129.736	91148.892	91168.048	91187.204	91206.361	91225.518	91244.676	91263.834	531
532	91282.992	91302.151	91321.310	91340.469	91359.629	91378.789	91397.950	91417.110	91436.272	91455.433	532
533	91474.595	91493.758	91512.921	91532.084	91551.247	91570.411	91589.575	91608.740	91627.905	91647.071	533
534	91666.236	91685.402	91704.569	91723.736	91742.903	91762.071	91781.239	91800.407	91819.576	91838.745	534
535	91857.915	91877.085	91896.255	91915.425	91934.596	91953.768	91972.940	91992.112	92011.284	92030.457	535
536	92049.630	92068.804	92087.978	92107.152	92126.327	92145.502	92164.678	92183.854	92203.030	92222.206	536
537	92241.383	92260.561	92279.738	92298.917	92318.095	92337.274	92356.453	92375.633	92394.813	92413.993	537
538	92433.174	92452.355	92471.536	92490.718	92509.900	92529.083	92548.266	92567.449	92586.633	92605.817	538
539	92625.001	92644.186	92663.371	92682.557	92701.742	92720.929	92740.115	92759.302	92778.490	92797.678	539
540	92816.866	92836.054	92855.243	92874.432	92893.622	92912.812	92932.002	92951.193	92970.384	92989.575	540
541	93008.767	93027.959	93047.152	93066.345	93085.538	93104.732	93123.926	93143.120	93162.315	93181.510	541
542	93200.706	93219.902	93239.098	93258.295	93277.492	93296.689	93315.887	93335.085	93354.283	93373.482	542
543	93392.681	93411.881	93431.081	93450.281	93469.482	93488.683	93507.884	93527.086	93546.288	93565.491	543
544	93584.694	93603.897	93623.100	93642.304	93661.509	93680.713	93699.919	93719.124	93738.330	93757.536	544
545	93776.743	93795.950	93815.157	93834.364	93853.573	93872.781	93891.990	93911.199	93930.408	93949.618	545
546	93968.828	93988.039	94007.250	94026.461	94045.673	94064.885	94084.097	94103.310	94122.523	94141.737	546
547	94160.951	94180.165	94199.380	94218.595	94237.810	94257.026	94276.242	94295.458	94314.675	94333.892	547
548	94353.110	94372.328	94391.546	94410.765	94429.984	94449.203	94468.423	94487.643	94506.863	94526.084	548
549	94545.305	94564.527	94583.749	94602.971	94622.194	94641.417	94660.640	94679.864	94699.088	94718.312	549
550	94737.537	94756.762	94775.988	94795.214	94814.440	94833.667	94852.894	94872.121	94891.349	94910.577	550
551	94929.805	94949.034	94968.263	94987.493	95006.723	95025.953	95045.184	95064.415	95083.646	95102.878	551
552	95122.110	95141.342	95160.575	95179.808	95199.042	95218.276	95237.510	95256.745	95275.980	95295.215	552
553	95314.451	95333.687	95352.923	95372.160	95391.397	95410.635	95429.872	95449.111	95468.349	95487.588	553
554	95506.827	95526.067	95545.307	95564.548	95583.788	95603.029	95622.271	95641.513	95660.755	95679.998	554
555	95699.241	95718.484	95737.727	95756.971	95776.216	95795.461	95814.706	95833.951	95853.197	95872.443	555
556	95891.690	95910.936	95930.184	95949.431	95968.679	95987.928	96007.176	96026.425	96045.675	96064.925	556
557	96084.175	96103.425	96122.676	96141.927	96161.179	96180.431	96199.683	96218.936	96238.189	96257.442	557
558	96276.696	96295.950	96315.204	96334.459	96353.714	96372.970	96392.225	96411.482	96430.738	96449.995	558
559	96469.252	96488.510	96507.768	96527.026	96546.285	96565.544	96584.804	96604.063	96623.324	96642.584	559
560	96661.845	96681.106	96700.368	96719.630	96738.892	96758.155	96777.418	96796.681	96815.945	96835.209	560

n	0	1	2	3	4	5	6	7	8	9	n
561	96854.473	96873.738	96893.003	96912.269	96931.535	96950.801	96970.067	96989.334	97008.602	97027.869	561
562	97047.137	97066.406	97085.674	97104.943	97124.213	97143.482	97162.753	97182.023	97201.294	97220.565	562
563	97239.837	97259.109	97278.381	97297.653	97316.926	97336.200	97355.473	97374.747	97394.022	97413.297	563
564	97432.572	97451.847	97471.123	97490.399	97509.676	97528.953	97548.230	97567.507	97586.785	97606.064	564
565	97625.342	97644.621	97663.901	97683.180	97702.460	97721.741	97741.022	97760.303	97779.584	97798.866	565
566	97818.148	97837.431	97856.714	97875.997	97895.280	97914.564	97933.849	97953.133	97972.418	97991.704	566
567	98010.989	98030.275	98049.562	98068.849	98088.136	98107.423	98126.711	98145.999	98165.288	98184.577	567
568	98203.866	98223.155	98242.445	98261.736	98281.026	98300.317	98319.609	98338.900	98358.192	98377.485	568
569	98396.778	98416.071	98435.364	98454.658	98473.952	98493.247	98512.542	98531.837	98551.132	98570.428	569
570	98589.725	98609.021	98628.318	98647.615	98666.913	98686.211	98705.509	98724.808	98744.107	98763.407	570
571	98782.706	98802.007	98821.307	98840.608	98859.909	98879.211	98898.512	98917.815	98937.117	98956.420	571
572	98975.723	98995.027	99014.331	99033.635	99052.940	99072.245	99091.550	99110.856	99130.162	99149.469	572
573	99168.775	99188.083	99207.390	99226.698	99246.006	99265.314	99284.623	99303.933	99323.242	99342.552	573
574	99361.862	99381.173	99400.484	99419.795	99439.107	99458.419	99477.731	99497.044	99516.357	99535.670	574
575	99554.984	99574.298	99593.612	99612.927	99632.242	99651.558	99670.874	99690.190	99709.506	99728.823	575
576	99748.140	99767.458	99786.776	99806.094	99825.413	99844.732	99864.051	99883.371	99902.691	99922.011	576
577	99941.332	99960.653	99979.974	99999.296	100018.618	100037.940	100057.263	100076.586	100095.910	100115.233	577
578	100134.558	100153.882	100173.207	100192.532	100211.858	100231.183	100250.510	100269.836	100289.163	100308.490	578
579	100327.818	100347.146	100366.474	100385.803	100405.132	100424.461	100443.791	100463.121	100482.451	100501.782	579
580	100521.113	100540.444	100559.776	100579.108	100598.441	100617.773	100637.107	100656.440	100675.774	100695.108	580
581	100714.443	100733.777	100753.113	100772.448	100791.784	100811.120	100830.457	100849.794	100869.131	100888.468	581
582	100907.806	100927.145	100946.483	100965.822	100985.162	101004.501	101023.841	101043.182	101062.522	101081.863	582
583	101101.205	101120.546	101139.888	101159.231	101178.574	101197.917	101217.260	101236.604	101255.948	101275.292	583
584	101294.637	101313.982	101333.328	101352.674	101372.020	101391.366	101410.713	101430.060	101449.408	101468.756	584
585	101488.104	101507.453	101526.802	101546.151	101565.500	101584.850	101604.201	101623.551	101642.902	101662.254	585
586	101681.605	101700.957	101720.309	101739.662	101759.015	101778.368	101797.722	101817.076	101836.431	101855.785	586
587	101875.140	101894.496	101913.851	101933.207	101952.564	101971.921	101991.278	102010.635	102029.993	102049.351	587
588	102068.710	102088.068	102107.427	102126.787	102146.147	102165.507	102184.867	102204.228	102223.589	102242.951	588
589	102262.313	102281.675	102301.037	102320.400	102339.764	102359.127	102378.491	102397.855	102417.220	102436.585	589
590	102455.950	102475.316	102494.681	102514.048	102533.414	102552.781	102572.149	102591.516	102610.884	102630.252	590
591	102649.621	102668.990	102688.359	102707.729	102727.099	102746.469	102765.840	102785.211	102804.582	102823.954	591
592	102843.326	102862.698	102882.071	102901.444	102920.817	102940.191	102959.565	102978.940	102998.314	103017.689	592
593	103037.065	103056.440	103075.816	103095.193	103114.570	103133.947	103153.324	103172.702	103192.080	103211.458	593
594	103230.837	103250.216	103269.596	103288.975	103308.356	103327.736	103347.117	103366.498	103385.879	103405.261	594
595	103424.643	103444.026	103463.409	103482.792	103502.175	103521.559	103540.943	103560.328	103579.712	103599.098	595
596	103618.483	103637.869	103657.255	103676.641	103696.028	103715.415	103734.803	103754.191	103773.579	103792.967	596
597	103812.356	103831.745	103851.135	103870.525	103889.915	103909.305	103928.696	103948.088	103967.479	103986.871	597
598	104006.263	104025.656	104045.048	104064.442	104083.835	104103.229	104122.623	104142.018	104161.413	104180.808	598
599	104200.203	104219.599	104238.995	104258.392	104277.789	104297.186	104316.583	104335.981	104355.380	104374.778	599
600	104394.177	104413.576	104432.976	104452.375	104471.776	104491.176	104510.577	104529.978	104549.380	104568.782	600
601	104588.184	104607.586	104626.989	104646.392	104665.796	104685.200	104704.604	104724.009	104743.413	104762.819	601
602	104782.224	104801.630	104821.036	104840.443	104859.849	104879.257	104898.664	104918.072	104937.480	104956.889	602
603	104976.298	104995.707	105015.116	105034.526	105053.936	105073.347	105092.758	105112.169	105131.580	105150.992	603
604	105170.404	105189.817	105209.229	105228.643	105248.056	105267.470	105286.884	105306.299	105325.713	105345.128	604
605	105364.544	105383.960	105403.376	105422.792	105442.209	105461.626	105481.044	105500.461	105519.879	105539.298	605
606	105558.717	105578.136	105597.555	105616.975	105636.395	105655.815	105675.236	105694.657	105714.079	105733.500	606
607	105752.922	105772.345	105791.768	105811.191	105830.614	105850.038	105869.462	105888.886	105908.311	105927.736	607
608	105947.161	105966.587	105986.013	106005.439	106024.866	106044.293	106063.720	106083.148	106102.576	106122.004	608
609	106141.433	106160.862	106180.291	106199.721	106219.151	106238.581	106258.012	106277.443	106296.874	106316.305	609
610	106335.737	106355.170	106374.602	106394.035	106413.468	106432.902	106452.336	106471.770	106491.205	106510.639	610
611	106530.075	106549.510	106568.946	106588.382	106607.819	106627.256	106646.693	106666.130	106685.568	106705.006	611
612	106724.445	106743.883	106763.323	106782.762	106802.202	106821.642	106841.082	106860.523	106879.964	106899.406	612
613	106918.847	106938.289	106957.732	106977.174	106996.617	107016.061	107035.505	107054.949	107074.393	107093.838	613
614	107113.283	107132.728	107152.174	107171.620	107191.066	107210.512	107229.959	107249.407	107268.854	107288.302	614
615	107307.750	107327.199	107346.648	107366.097	107385.547	107404.997	107424.447	107443.897	107463.348	107482.799	615
616	107502.251	107521.703	107541.155	107560.607	107580.060	107599.513	107618.967	107638.420	107657.875	107677.329	616
617	107696.784	107716.239	107735.694	107755.150	107774.606	107794.062	107813.519	107832.976	107852.433	107871.891	617
618	107891.349	107910.807	107930.266	107949.725	107969.184	107988.644	108008.104	108027.564	108047.025	108066.485	618
619	108085.947	108105.408	108124.870	108144.332	108163.795	108183.258	108202.721	108222.184	108241.648	108261.112	619
620	108280.577	108300.041	108319.506	108338.972	108358.438	108377.904	108397.370	108416.837	108436.304	108455.771	620
621	108475.239	108494.707	108514.175	108533.644	108553.113	108572.582	108592.052	108611.522	108630.992	108650.462	621
622	108669.933	108689.404	108708.876	108728.348	108747.820	108767.293	108786.765	108806.238	108825.712	108845.186	622
623	108864.660	108884.134	108903.609	108923.084	108942.559	108962.035	108981.511	109000.988	109020.464	109039.941	623
624	109059.418	109078.896	109098.374	109117.852	109137.331	109156.810	109176.289	109195.769	109215.249	109234.729	624
625	109254.209	109273.690	109293.171	109312.653	109332.134	109351.617	109371.099	109390.582	109410.065	109429.548	625
626	109449.032	109468.516	109488.000	109507.485	109526.970	109546.455	109565.941	109585.427	109604.913	109624.400	626
627	109643.887	109663.374	109682.861	109702.349	109721.837	109741.326	109760.815	109780.304	109799.793	109819.283	627
628	109838.773	109858.264	109877.754	109897.245	109916.737	109936.228	109955.720	109975.213	109994.705	110014.198	628
629	110033.692	110053.185	110072.679	110092.173	110111.668	110131.163	110150.658	110170.153	110189.649	110209.145	629
630	110228.642	110248.139	110267.636	110287.133	110306.631	110326.129	110345.627	110365.126	110384.625	110404.124	630

Table of 2n ln n *for values* of n *from 1 to 10,000*—Continued

	0	1	2	3	4	5	6	7	8	9	
631	110423.624	110443.124	110462.624	110482.125	110501.625	110521.127	110540.628	110560.130	110579.632	110599.135	631
632	110618.637	110638.141	110657.644	110677.148	110696.652	110716.156	110735.661	110755.166	110774.671	110794.177	632
633	110813.683	110833.189	110852.696	110872.203	110891.710	110911.217	110930.725	110950.233	110969.742	110989.251	633
634	111008.760	111028.269	111047.779	111067.289	111086.799	111106.310	111125.821	111145.332	111164.844	111184.356	624
635	111203.868	111223.381	111242.894	111262.407	111281.920	111301.434	111320.948	111340.463	111359.978	111379.493	635
636	111399.008	111418.524	111438.040	111457.556	111477.073	111496.590	111516.107	111535.625	111555.143	111574.661	636
637	111594.179	111613.698	111633.217	111652.737	111672.257	111691.777	111711.297	111730.818	111750.339	111769.861	637
638	111789.382	111808.904	111828.427	111847.949	111867.472	111886.995	111906.519	111926.043	111945.567	111965.092	638
629	111984.616	112004.141	112023.667	112043.193	112062.719	112082.245	112101 772	112121.299	112140.826	112160.354	639
640	112179.882	112199.410	112218.939	112238.468	112257.997	112277.526	112297.056	112316.586	112336.117	112355.647	640
641	112375.178	112394.710	112414.242	112433.774	112453.306	112472.839	112492.371	112511.905	112531.438	112550.972	641
642	112570.506	112590.041	112609.576	112629.111	112648.646	112668.182	112687.718	112707.254	112726.791	112746.328	642
643	112765.865	112785.403	112804.941	112824.479	112844.018	112863.557	112883.096	112902.635	112922.175	112941.715	643
644	112961.256	112980.796	113000.337	113019.879	113039.420	113058.962	113078.505	113098.047	113117.590	113137.133	644
645	113156.677	113176.221	113195.765	113215.309	113234.854	113254.399	113273.944	113293.490	113313.036	113332.582	645
646	113352.129	113371.676	113391.223	113410.771	113430.318	113449.867	113469.415	113488.964	113508.513	113528.062	646
647	113547.612	113567.162	113586.712	113606.263	113625.814	113645.365	113664.917	113684.469	113704.021	113723.573	647
648	113743.126	113762.679	113782.233	113801.786	113821.340	113840.895	113860.449	113880.004	113899.560	113919.115	648
649	113938.671	113958.227	113977.784	113997.341	114016.898	114036.455	114056.013	114075.571	114095.129	114114.688	649
650	114134.247	114153.806	114173.366	114192.926	114212.486	114232.046	114251.607	114271.168	114290.730	114310.291	650
651	114329.853	114349.416	114368.978	114388.541	114408.105	114427.668	114447.232	114466.796	114486.361	114505.926	651
652	114525.491	114545.056	114564.622	114584.188	114603.754	114623.321	114642.888	114662.455	114682.023	114701.590	652
653	114721.159	114740.727	114760.296	114779.865	114799.434	114819.004	114838.574	114858.144	114877.715	114897.286	653
654	114916.857	114936.429	114956.000	114975.573	114995.145	115014.718	115034.291	115053.864	115073.438	115093.012	654
655	115112.586	115132.161	115151.736	115171.311	115190.886	115210.462	115230.038	115249.615	115269.192	115288.769	655
656	115308.346	115327.924	115347.501	115367.080	115386.658	115406.237	115425.816	115445.396	115464.976	115484.556	656
657	115504.136	115523.717	115543.298	115562.879	115582.461	115602.043	115621.625	115641.207	115660.790	115680.373	657
658	115699.957	115719.540	115739.124	115758.709	115778.293	115797.878	115817.464	115837.049	115856.635	115876.221	658
659	115895.808	115915.394	115934.981	115954.569	115974.157	115993.745	116013.333	116032.921	116052.510	116072.099	659
660	116091.689	116111.279	116130.869	116150.459	116170.050	116189.641	116209.232	116228.824	116248.416	116268.008	660
661	116287.601	116307.193	116326.787	116346.380	116365.974	116385.568	116405.162	116424.757	116444.352	116463.947	661
662	116483.543	116503.138	116522.735	116542.331	116561.928	116581.525	116601.122	116620.720	116640.318	116659.916	662
663	116679.515	116699.114	116718.713	116738.312	116757.912	116777.512	116797.112	116816.713	116836.314	116855.915	663
664	116875.517	116895.119	116914.721	116934.323	116953.926	116973.529	116993.133	117012.736	117032.340	117051.945	664
665	117071.549	117091.154	117110.759	117130.365	117149.971	117169.577	117189.183	117208.790	117228.397	117248.004	665
666	117267.612	117287.220	117306.828	117326.436	117346.045	117365.654	117385.264	117404.873	117424.483	117444.094	666
667	117463.704	117483.315	117502.926	117522.538	117542.150	117561.762	117581.374	117600.987	117620.600	117640.213	667
668	117659.827	117679.441	117699.055	117718.669	117738.284	117757.899	117777.515	117797.130	117816.746	117836.363	668
669	117855.979	117875.596	117895.213	117914.831	117934.449	117954.067	117973.685	117993.304	118012.923	118032.542	669
670	118052.162	118071.781	118091.402	118111.022	118130.643	118150.264	118169.885	118189.507	118209.129	118228.751	670
671	118248.374	118267.997	118287.620	118307.243	118326.867	118346.491	118366.115	118285.740	118405.365	118424.990	671
672	118444.616	118464.242	118483.868	118503.494	118523.121	118542.748	118562.375	118582.003	118601.631	118621.259	672
673	118640.887	118660.516	118680.145	118699.775	118719.404	118739.034	118758.665	118778.295	118797.926	118817.557	673
674	118837.189	118856.821	118876.453	118896.085	118915.718	118935.351	118954.984	118974.618	118994.251	119013.886	674
675	119033.520	119053.155	119072.790	119092.425	119112.061	119131.697	119151.333	119170.969	119190.606	119210.243	675
676	119229.881	119249.518	119269.156	119288.795	119308.433	119328.072	119347.711	119367.351	119386.991	119406.631	676
677	119426.271	119445.912	119465.553	119485.194	119504.836	119524.477	119544.120	119563.762	119583.405	119603.048	677
678	119622.691	119642.335	119661.979	119681.623	119701.267	119720.912	119740.557	119760.203	119779.848	119799.494	678
679	119819.140	119838.787	119858.434	119878.081	119897.728	119917.376	119937.024	119956.673	119976.321	119995.970	679
680	120015.619	120035.269	120054.919	120074.569	120094.219	120113.870	120133.521	120153.172	120172.824	120192.475	680
681	120212.128	120231.780	120251.433	120271.086	120290.739	120310.393	120330.047	120349.701	120369.355	120389.010	681
682	120408.665	120428.321	120447.976	120467 632	120487.288	120506.945	120526.602	120546.259	120565.916	120585.574	682
683	120605.232	120624.890	120644.549	120664.208	120683.867	120703.527	120723.186	120742.846	120762.507	120782.167	683
684	120801.828	120821.490	120841.151	120860.813	120880.475	120900.137	120919.800	120939.463	120959.126	120978.790	684
685	120998.454	121018.118	121037.782	121057.447	121077.112	121096.777	121116.443	121136.109	121155.775	121175.442	685
686	121195.108	121214.776	121234.443	121254.111	121273.779	121293.447	121313.115	121332.784	121352.453	121372.123	686
687	121391.792	121411.462	121431.133	121450.803	121470.474	121490.145	121509.817	121529.488	121549.160	121568.833	687
688	121588.505	121608.178	121627.851	121647.525	121667.199	121686.873	121706.547	121726.222	121745.897	121765.572	688
689	121785.247	121804.923	121824.599	121844.276	121863.952	121883.629	121903.306	121922.984	121942.662	121962.340	689
690	121982.018	122001.697	122021.376	122041.055	122060.735	122080.415	122100.095	122119.775	122139.456	122159.137	690
691	122178.818	122198.500	122218.182	122237.864	122257.546	122277.229	122296.912	122316.596	122336.279	122355.963	691
692	122375.647	122395.332	122415.017	122434.702	122454.387	122474.073	122493.759	122513.445	122533.131	122552.818	692
693	122572.505	122592.192	122611.880	122631.568	122651.256	122670.945	122690.634	122710.323	122730.012	122749.702	693
694	122769.392	122789.082	122808.773	122828.463	122848.155	122867.846	122887.538	122907.230	122926.922	122946.615	694
695	122966.307	122986.001	123005.694	123025.388	123045.082	123064.776	123084.471	123104.165	123123.861	123143.556	695
696	123163.252	123182.948	123202.644	123222.341	123242.037	123261.735	123281.432	123301.130	123320.828	123340.526	696
697	123360.225	123379.924	123399.623	123419.322	123439.022	123458.722	123478.422	123498.123	123517.824	123537.525	697
698	123557.227	123576.928	123596.630	123616.333	123636.035	123655.738	123675.441	123695.145	123714.849	123734.553	698
699	123754.257	123773.962	123793.666	123813.372	123833.077	123852.783	123872.489	123892.195	123911.902	123931.609	699
700	123951.316	123971.023	123990.731	124010.439	124030.148	124049.856	124069.565	124089.274	124108.984	124128.693	700

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701	124148.404	124168.114	124187.824	124207.535	124227.247	124246.958	124266.670	124286.382	124306.094	124325.807	701
702	124345.520	124365.233	124384.946	124404.660	124424.374	124444.088	124463.803	124483.518	124503.233	124522.948	702
703	124542.664	124562.380	124582.097	124601.813	124621.530	124641.247	124660.965	124680.682	124700.400	124720.119	703
704	124739.837	124759.556	124779.275	124798.995	124818.714	124838.434	124858.155	124877.875	124897.596	124917.317	704
705	124937.039	124956.760	124976.482	124996.205	125015.927	125035.650	125055.373	125075.097	125094.820	125114.544	705
706	125134.269	125153.993	125173.718	125193.443	125213.168	125232.894	125252.620	125272.346	125292.073	125311.800	706
707	125331.527	125351.254	125370.982	125390.710	125410.438	125430.166	125449.895	125469.624	125489.354	125509.083	707
708	125528.813	125548.543	125568.274	125588.005	125607.736	125627.467	125647.199	125666.931	125686.663	125706.395	708
709	125726.128	125745.861	125765.594	125785.328	125805.062	125824.796	125844.530	125864.265	125884.000	125903.735	709
710	125923.471	125943.207	125962.943	125982.679	126002.416	126022.153	126041.890	126061.628	126081.365	126101.104	710
711	126120.842	126140.581	126160.319	126180.059	126199.798	126219.538	126239.278	126259.018	126278.759	126298.500	711
712	126318.241	126337.983	126357.724	126377.466	126397.209	126416.951	126436.694	126456.437	126476.181	126495.924	712
713	126515.668	126535.413	126555.157	126574.902	126594.647	126614.393	126634.138	126653.884	126673.630	126693.377	713
714	126713.124	126732.871	126752.618	126772.366	126792.114	126811.862	126831.610	126851.359	126871.108	126890.858	714
715	126910.607	126930.357	126950.107	126969.858	126989.608	127009.359	127029.111	127048.862	127068.614	127088.366	715
716	127108.118	127127.871	127147.624	127167.377	127187.131	127206.885	127226.639	127246.393	127266.148	127285.903	716
717	127305.658	127325.413	127345.169	127364.925	127384.681	127404.438	127424.195	172443.952	127463.709	127483.467	717
718	127503.225	127522.983	127542.742	127562.500	127582.260	127602.019	127621.779	127641.538	127661.299	127681.059	718
719	127700.820	127720.581	127740.342	127760.104	127779.866	127799.628	127819.390	127839.153	127858.916	127878.679	719
720	127898.443	127918.207	127937.971	127957.735	127977.500	127997.265	128017.030	128036.795	128056.561	128076.327	720
721	128096.093	128115.860	128135.627	128155.394	128175.161	128194.929	128214.697	128234.465	128254.234	128274.003	721
722	128293.772	128313.541	128333.311	128353.081	128372.851	128392.621	128412.392	128432.163	128451.934	128471.706	722
723	128491.478	128511.250	128531.022	128550.795	128570.568	128590.341	128610.115	128629.888	128649.662	128669.437	723
724	128689.211	128708.986	128728.761	128748.537	128768.313	128788.089	128807.865	128827.641	128847.418	128867.195	724
725	128886.973	128906.750	128926.528	128946.307	128966.085	128985.864	129005.643	129025.422	129045.202	129064.982	725
726	129084.762	129104.542	129124.323	129144.104	129163.885	129183.666	129203.448	129223.230	129243.013	129262.795	726
727	129282.578	129302.361	129322.145	129341.928	129361.712	129381.497	129401.281	129421.066	129440.851	129460.636	727
728	129480.422	129500.208	129519.994	129539.781	129559.567	129579.354	129599.142	129618.929	129638.717	129658.505	728
729	129678.294	129698.082	129717.871	129737.660	129757.450	129777.240	129797.030	129816.820	129836.610	129856.401	729
730	129876.192	129895.984	129915.776	129935.567	129955.360	129975.152	129994.945	130014.738	130034.531	130054.325	730
731	130074.119	130093.913	130113.707	130133.502	130153.297	130173.092	130192.888	130212.683	130232.479	130252.276	731
732	130272.072	130291.869	130311.666	130331.464	130351.261	130371.059	130390.858	130410.656	130430.455	130450.254	732
733	130470.053	130489.853	130509.653	130529.453	130549.253	130569.054	130588.855	130608.656	130628.458	130648.260	733
734	130668.062	130687.864	130707.667	130727.469	130747.273	130767.076	130786.880	130806.684	130826.488	130846.292	734
735	130866.097	130885.902	130905.707	130925.513	130945.319	130965.125	130984.931	131004.738	131024.545	131044.352	735
736	131064.160	131083.968	131103.776	131123.584	131143.393	131163.201	131183.010	131202.820	131222.630	131242.439	736
737	131262.250	131282.060	131301.871	131321.682	131341.493	131361.305	131381.117	131400.929	131420.741	131440.554	737
738	131460.367	131480.180	131499.993	131519.807	131539.621	131559.435	131579.250	131599.065	131618.880	131638.695	738
739	131658.511	131678.327	131698.143	131717.959	131737.776	131757.593	131777.410	131797.228	131817.046	131836.864	739
740	131856.682	131876.501	131896.319	131916.139	131935.958	131955.778	131975.598	131995.418	132015.238	132035.059	740
741	132054.880	132074.702	132094.523	132114.345	132134.167	132153.989	132173.812	132193.635	132213.458	132233.282	741
742	132253.105	132272.929	132292.754	132312.578	132332.403	132352.228	132372.053	132391.879	132411.705	132431.531	742
743	132451.358	132471.184	132491.011	132510.838	132530.666	132550.494	132570.322	132590.150	132609.979	132629.808	743
744	132649.637	132669.466	132689.296	132709.126	132728.956	132748.786	132768.617	132788.448	132808.279	132828.111	744
745	132847.943	132867.775	132887.607	132907.440	132927.273	132947.106	132966.939	132986.773	133006.607	133026.441	745
746	133046.275	133066.110	133085.945	133105.781	133125.616	133145.452	133165.288	133185.124	133204.961	133224.798	746
747	133244.635	133264.472	133284.310	133304.148	133323.986	133343.825	133363.664	133383.503	133403.342	133423.182	747
748	133443.021	133462.862	133482.702	133502.543	133522.383	133542.225	133562.066	133581.908	133601.750	133621.592	748
749	133641.435	133661.277	133681.120	133700.964	133720.807	133740.651	133760.495	133780.340	133800.184	133820.029	749
750	133839.874	133859.720	133879.566	133899.412	133919.258	133939.104	133958.951	133978.798	133998.645	134018.493	750
751	134038.341	134058.189	134078.037	134097.886	134117.735	134137.584	134157.434	134177.283	134197.133	134216.983	751
752	134236.834	134256.685	134276.536	134296.387	134316.239	134336.091	134355.943	134375.795	134395.648	134415.501	752
753	134435.354	134455.207	134475.061	134494.915	134514.679	134534.624	134554.478	134574.333	134594.189	134614.044	753
754	134633.900	134653.756	134673.612	134693.469	134713.326	134733.183	134753.041	134772.898	134792.756	134812.614	754
755	134832.473	134852.332	134872.191	134892.050	134911.909	134931.769	134951.629	134971.490	134991.350	135011.211	755
756	135031.072	135050.933	135070.795	135090.657	135110.519	135130.382	135150.244	135170.107	135189.971	135209.834	756
757	135229.698	135249.562	135269.426	135289.291	135309.156	135329.021	135348.886	135368.752	135388.617	135408.484	757
758	135428.350	135448.217	135468.084	135487.951	135507.818	135527.686	135547.554	135567.422	135587.291	135607.160	758
759	135627.029	135646.898	135666.767	135686.637	135706.507	135726.378	135746.248	135766.119	135785.990	135805.862	759
760	135825.733	135845.605	135865.478	135885.350	135905.223	135925.096	135944.969	135964.843	135984.716	136004.590	760
761	136024.465	136044.339	136064.214	136084.089	136103.965	136123.840	136143.716	136163.592	136183.469	136203.345	761
762	136223.222	136243.099	136262.977	136282.855	136302.733	136322.611	136342.489	136362.368	136382.247	136402.126	762
763	136422.006	136441.886	136461.766	136481.646	136501.527	136521.408	136541.289	136561.170	136581.052	136600.934	763
764	136620.816	136640.698	136660.581	136680.464	136700.347	136720.231	136741.114	136759.998	136779.883	136799.767	764
765	136819.652	136839.537	136859.422	136879.308	136899.194	136919.080	136938.966	136958.853	136978.740	136998.627	765
766	137018.514	137038.402	137058.290	137078.178	137098.067	137117.955	137137.844	137157.734	137177.623	137197.513	766
767	137217.403	137237.293	137257.184	137277.074	137296.965	137316.857	137336.748	137356.640	137376.532	137396.425	767
768	137416.317	137436.210	137456.103	137475.997	137495.890	137515.784	137535.678	137555.573	137575.468	137595.363	768
769	137615.258	137635.153	137655.049	137674.945	137694.841	137714.738	137734.635	137754.532	137774.429	137794.326	769
770	137814.224	137834.122	137854.021	137873.919	137893.818	137913.717	137933.617	137953.516	137973.416	137993.316	770

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771	138013.217	138033.117	138053.018	138072.920	138092.821	138112.723	138132.625	138152.527	138172.429	138192.332	771
772	138212.235	138232.138	138252.042	138271.946	138291.850	138311.754	138331.659	138351.564	138371.469	138391.374	772
773	138411.280	138431.185	138451.092	138470.998	138490.905	138510.811	138530.719	138550.626	138570.534	138590.442	773
774	138610.350	138630.258	138650.167	138670.076	138689.985	138709.895	138729.804	138749.714	138769.625	138789.535	774
775	138809.446	138829.357	138849.268	138869.180	138889.091	138909.004	138928.916	138948.828	138968.741	138988.654	775
776	139008.568	139028.481	139048.395	139068.309	139088.224	139108.138	139128.053	139147.968	139167.884	139187.779	776
777	139207.715	139227.631	139247.548	139267.465	139287.382	139307.299	139327.216	139347.134	139367.052	139386.970	777
778	139406.889	139426.807	139446.726	139466.646	139486.565	139506.485	139526.405	139546.325	139566.246	139586.167	778
779	139606.088	139626.009	139645.931	139665.852	139685.775	139705.697	139725.620	139745.542	139765.466	139785.389	779
780	139805.313	139825.236	139845.161	139865.085	139885.010	139904.935	139924.860	139944.785	139964.711	139984.637	780
781	140004.563	140024.489	140044.416	140064.343	140084.270	140104.198	140124.125	140144.053	140163.982	140183.910	781
782	140203.839	140223.768	140243.697	140263.627	140283.556	140303.487	140323.417	140343.347	140363.278	140383.209	782
783	140403.140	140423.072	140443.004	140462.936	140482.868	140502.801	140522.734	140542.667	140562.600	140582.534	783
784	140602.468	140622.402	140642.336	140662.271	140682.206	140702.141	140722.076	140742.012	140761.948	140781.884	784
785	140801.820	140821.757	140841.694	140861.631	140881.568	140901.506	140921.444	140941.382	140961.321	140981.259	785
786	141001.198	141021.138	141041.077	141061.017	141080.957	141100.897	141120.837	141140.778	141160.719	141180.660	786
787	141200.602	141220.544	141240.486	141260.428	141280.370	141300.313	141320.256	141340.200	141360.143	141380.087	787
788	141400.031	141419.975	141439.920	141459.865	141479.810	141499.755	141519.700	141539.646	141559.592	141579.539	788
789	141599.485	141619.432	141639.379	141659.326	141679.274	141699.222	141719.170	141739.118	141759.067	141779.016	789
790	141798.965	141818.914	141838.864	141858.814	141878.764	141898.714	141918.665	141938.616	141958.567	141978.518	790
791	141998.470	142018.422	142038.374	142058.326	142078.279	142098.232	142118.185	142138.138	142158.092	142178.046	791
792	142198.000	142217.955	142237.909	142257.864	142277.819	142297.775	142317.731	142337.686	142357.643	142377.599	792
793	142397.556	142417.513	142437.470	142457.427	142477.385	142497.343	142517.301	142537.260	142557.218	142577.177	793
794	142597.137	142617.096	142637.056	142657.016	142676.976	142696.936	142716.897	142736.858	142756.819	142776.781	794
795	142796.743	142816.704	142836.667	142856.629	142876.592	142896.555	142916.518	142936.482	142956.445	142976.409	795
796	142996.374	143016.338	143036.303	143056.268	143076.233	143096.199	143116.164	143136.130	143156.097	143176.063	796
797	143196.030	143215.997	143235.964	143255.932	143275.899	143295.867	143315.836	143335.804	143355.773	143375.742	797
798	143395.711	143415.681	143435.650	143455.620	143475.591	143495.561	143515.532	143535.503	143555.474	143575.446	798
799	143595.418	143615.390	143635.362	143655.334	143675.307	143695.280	143715.253	143735.227	143755.201	143775.175	799
800	143795.149	143815.124	143835.098	143855.073	143875.049	143895.024	143915.000	143934.976	143954.952	143974.929	800
801	143994.905	144014.882	144034.860	144054.837	144074.815	144094.793	144114.771	144134.750	144154.729	144174.708	801
802	144194.687	144214.666	144234.646	144254.626	144274.606	144294.587	144314.568	144334.549	144354.530	144374.511	802
803	144394.493	144414.475	144434.457	144454.440	144474.423	144494.406	144514.389	144534.372	144554.356	144574.340	803
804	144594.324	144614.309	144634.294	144654.279	144674.264	144694.249	144714.235	144734.221	144754.207	144774.194	804
805	144794.181	144814.168	144834.155	144854.142	144874.130	144894.118	144914.106	144934.095	144954.083	144974.072	805
806	144994.062	145014.051	145034.041	145054.031	145074.021	145094.011	145114.002	145133.993	145153.984	145173.976	806
807	145193.967	145213.959	145233.951	145253.944	145273.937	145293.929	145313.923	145333.916	145353.910	145373.904	807
808	145393.898	145413.892	145433.887	145453.882	145473.877	145493.872	145513.868	145533.864	145553.860	145573.857	808
809	145593.853	145613.850	145633.847	145653.845	145673.842	145693.840	145713.838	145733.837	145753.835	145773.834	809
810	145793.833	145813.833	145833.832	145853.832	145873.832	145893.832	145913.833	145933.834	145953.835	145973.836	810
811	145993.838	146013.840	146033.842	146053.844	146073.847	146093.850	146113.853	146133.856	146153.859	146173.863	811
812	146193.867	146213.872	146233.876	146253.881	146273.886	146293.891	146313.897	146333.903	146353.909	146373.915	812
813	146393.921	146413.928	146433.935	146453.942	146473.950	146493.958	146513.966	146533.974	146553.982	146573.991	813
814	146594.000	146614.009	146634.019	146654.028	146674.038	146694.048	146714.059	146734.070	146754.081	146774.092	814
815	146794.103	146814.115	146834.127	146854.139	146874.151	146894.164	146914.177	146934.190	146954.203	146974.217	815
816	146994.231	147014.245	147034.259	147054.274	147074.289	147094.304	147114.319	147134.335	147154.351	147174.367	816
817	147194.383	147214.400	147234.416	147254.434	147274.451	147294.468	147314.486	147334.504	147354.523	147374.541	817
818	147394.560	147414.579	147434.598	147454.618	147474.637	147494.657	147514.678	147534.698	147554.719	147574.740	818
819	147594.761	147614.782	147634.804	147654.826	147674.848	147694.871	147714.893	147734.916	147754.940	147774.963	819
820	147794.987	147815.010	147835.035	147855.059	147875.084	147895.109	147915.134	147935.159	147955.185	147975.210	820
821	147995.237	148015.263	148035.289	148055.316	148075.343	148095.371	148115.398	148135.426	148155.454	148175.482	821
822	148195.511	148215.540	148235.569	148255.598	148275.627	148295.657	148315.687	148335.717	148355.748	148375.779	822
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824	148596.133	148616.166	148636.200	148656.234	148676.269	148696.303	148716.338	148736.373	148756.408	148776.444	824
825	148796.480	148816.516	148836.552	148856.589	148876.625	148896.663	148916.700	148936.737	148956.775	148976.813	825
826	148996.851	149016.890	149036.928	149056.967	149077.007	149097.046	149117.086	149137.126	149157.166	149177.206	826
827	149197.247	149217.288	149237.329	149257.370	149277.412	149297.454	149317.496	149337.538	149357.581	149377.624	827
828	149397.667	149417.710	149437.754	149457.798	149477.842	149497.886	149517.930	149537.975	149558.020	149578.065	828
829	149598.111	149618.157	149638.203	149658.249	149678.295	149698.342	149718.389	149738.436	149758.484	149778.531	829
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831	149999.071	150019.122	150039.173	150059.224	150079.275	150099.326	150119.378	150139.430	150159.482	150179.535	831
832	150199.588	150219.641	150239.694	150259.747	150279.801	150299.855	150319.909	150339.963	150360.018	150380.073	832
833	150400.128	150420.183	150440.239	150460.295	150480.351	150500.407	150520.464	150540.521	150560.578	150580.635	833
834	150600.692	150620.750	150640.808	150660.866	150680.925	150700.984	150721.043	150741.102	150761.161	150781.221	834
835	150801.281	150821.341	150841.401	150861.462	150881.523	150901.584	150921.645	150941.707	150961.769	150981.831	835
836	151001.893	151021.956	151042.018	151062.081	151082.145	151102.208	151122.272	151142.336	151162.400	151182.465	836
837	151202.529	151222.594	151242.659	151262.725	151282.790	151302.856	151322.922	151342.989	151363.055	151383.122	837
838	151403.189	151423.257	151443.324	151463.392	151483.460	151503.528	151523.597	151543.666	151563.735	151583.804	838
839	151603.873	151623.943	151644.013	151664.083	151684.154	151704.224	151724.295	151744.366	151764.438	151784.509	839
840	151804.581	151824.653	151844.726	151864.798	151884.871	151904.944	151925.017	151945.091	151965.165	151985.239	840

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841	152005. 313	152025. 387	152045. 462	152065. 537	152085. 612	152105. 688	152125. 763	152145. 839	152165. 915	152185. 992	841
842	152206. 068	152226. 145	152246. 222	152266. 300	152286. 377	152306. 455	152326. 533	152346. 611	152366. 690	152386. 768	842
843	152406. 847	152426. 927	152447. 006	152467. 086	152487. 166	152507. 246	152527. 326	152547. 407	152567. 488	152587. 569	843
844	152607. 650	152627. 732	152647. 814	152667. 896	152687. 978	152708. 061	152728. 143	152748. 226	152768. 310	152788. 393	844
845	152808. 477	152828. 561	152848. 645	152868. 730	152888. 814	152908. 899	152928. 984	152949. 070	152969. 155	152989. 241	845
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847	153210. 201	153230. 290	153250. 379	153270. 468	153290. 557	153310. 647	153330. 737	153350. 827	153370. 917	153391. 008	847
848	153411. 099	153431. 190	153451. 281	153471. 373	153491. 464	153511. 556	153531. 648	153551. 741	153571. 834	153591. 927	848
849	153612. 020	153632. 113	153652. 207	153672. 301	153692. 395	153712. 489	153732. 584	153752. 679	153772. 774	153792. 869	849
850	153812. 964	153833. 060	153853. 156	153873. 252	153893. 349	153913. 446	153933. 543	153953. 640	153973. 737	153993. 835	850
851	154013. 933	154034. 031	154054. 129	154074. 228	154094. 326	154114. 426	154134. 525	154154. 624	154174. 724	154194. 824	851
852	154214. 924	154235. 025	154255. 125	154275. 226	154295. 328	154315. 429	154335. 531	154355. 632	154375. 735	154395. 837	852
853	154415. 939	154436. 042	154456. 145	154476. 249	154496. 352	154516. 456	154536. 560	154556. 664	154576. 768	154596. 873	853
854	154616. 978	154637. 083	154657. 189	154677. 294	154697. 400	154717. 506	154737. 612	154757. 719	154777. 826	154797. 933	854
855	154818. 040	154838. 148	154858. 255	154878. 363	154898. 471	154918. 580	154938. 689	154958. 797	154978. 907	154999. 016	855
856	155019. 126	155039. 235	155059. 345	155079. 456	155099. 566	155119. 677	155139. 788	155159. 899	155180. 011	155200. 122	856
857	155220. 234	155240. 346	155260. 459	155280. 571	155300. 684	155320. 797	155340. 911	155361. 024	155381. 138	155401. 252	857
858	155421. 366	155441. 481	155461. 596	155481. 711	155501. 826	155521. 941	155542. 057	155562. 173	155582. 289	155602. 405	858
859	155622. 522	155642. 639	155662. 756	155682. 873	155702. 991	155723. 108	155743. 226	155763. 345	155783. 463	155803. 582	859
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862	156226 128	156246. 252	156266. 376	156286. 500	156306. 624	156326. 749	156346. 874	156366. 999	156387. 125	156407. 250	862
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866	157031. 260	157051. 393	157071. 527	157091. 660	157111. 794	157131. 928	157152. 062	157172. 197	157192. 331	157212. 466	866
867	157232. 601	157252. 737	157272. 872	157293. 008	157313. 144	157333. 280	157353. 417	157373. 554	157393. 691	157413. 828	867
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875	158844. 157	158864. 311	158884. 465	158904. 619	158924. 773	158944. 928	158965. 083	158985. 238	159005. 393	159025. 549	875
876	159045. 705	159065. 861	159086. 017	159106. 173	159126. 330	159146. 487	159166. 644	159186. 802	159206. 959	159227. 117	876
877	159247. 275	159267. 433	159287. 592	159307. 751	159327. 910	159348. 069	159368. 228	159388. 388	159408. 548	159428. 708	877
878	159448. 868	159469. 029	159489. 190	159509. 351	159529. 512	159549. 673	159569. 835	159589. 997	159610. 159	159630. 322	878
879	159650. 484	159670. 647	159690. 810	159710. 974	159731. 137	159751. 301	159771. 465	159791. 629	159811. 794	159831. 958	879
880	159852. 123	159872. 288	159892. 454	159912. 619	159932. 785	159952. 951	159973. 117	159993. 284	160013. 450	160033. 617	880
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882	160255. 469	160275. 638	160295. 808	160315. 978	160336. 149	160356. 319	160376. 490	160396. 661	160416. 832	160437. 004	882
883	160457. 176	160477. 348	160497. 520	160517. 692	160537. 865	160558. 038	160578. 211	160598. 384	160618. 557	160638. 731	883
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887	161264. 229	161284. 410	161304. 592	161324. 773	161344. 955	161365. 137	161385. 319	161405. 501	161425. 684	161445. 866	887
888	161466. 049	161486. 233	161506. 416	161526. 600	161546. 784	161566. 968	161587. 152	161607. 337	161627. 521	161647. 706	888
889	161667. 892	161688. 077	161708. 263	161728. 449	161748. 635	161768. 821	161789. 008	161809. 195	161829. 382	161849. 569	889
890	161869. 757	161889. 944	161910. 132	161930. 320	161950. 509	161970. 697	161990. 886	162011. 075	162031. 265	162051. 454	890
891	162071. 644	162091. 834	162112. 024	162132. 215	162152. 405	162172. 596	162192. 787	162212. 978	162233. 170	162253. 362	891
892	162273. 554	162293. 746	162313. 938	162334. 131	162354. 324	162374. 517	162394. 710	162414. 904	162435. 098	162455. 292	892
893	162475. 486	162495. 680	162515. 875	162536. 070	162556. 265	162576. 460	162596. 656	162616. 852	162637. 048	162657. 244	893
894	162677. 441	162697. 637	162717. 834	162738. 031	162758. 229	162778. 426	162798. 624	162818. 822	162839. 020	162859. 219	894
895	162879. 418	162899. 617	162919. 816	162940. 015	162960. 215	162980. 414	163000. 615	163020. 815	163041. 015	163061. 216	895
896	163081. 417	163101. 618	163121. 819	163142. 021	163162. 223	163182. 425	163202. 627	163222. 830	163243. 032	163263. 235	896
897	163283. 439	163303. 642	163323. 846	163344. 049	163364. 254	163384. 458	163404. 662	163424. 867	163445. 072	163465. 277	897
898	163485. 483	163505. 688	163525. 894	163546. 100	163566. 306	163586. 513	163606. 720	163626. 927	163647. 134	163667. 341	898
899	163687. 549	163707. 757	163727. 965	163748. 173	163768. 382	163788. 590	163808. 799	163829. 008	163849. 218	163869. 427	899
900	163889. 637	163909. 847	163930. 058	163950. 268	163970. 479	163990. 690	164010. 901	164031. 112	164051. 324	164071. 536	900
901	164091. 748	164111. 960	164132. 173	164152. 386	164172. 599	164192. 812	164213. 025	164233. 239	164253. 453	164273. 667	901
902	164293. 881	164314. 095	164334. 310	164354. 525	164374. 740	164394. 956	164415. 171	164435. 387	164455. 603	164475. 819	902
903	164496. 036	164516. 253	164536. 470	164556. 687	164576. 904	164597. 122	164617. 340	164637. 558	164657. 776	164677. 994	903
904	164698. 213	164718. 432	164738. 651	164758. 871	164779. 090	164799. 310	164819. 530	164839. 750	164859. 971	164880. 192	904
905	164900. 413	164920. 634	164940. 855	164961. 077	164981. 298	165001. 521	165021. 743	165041. 965	165062. 188	165082. 411	905
906	165102. 634	165122. 857	165143. 081	165163. 305	165183. 529	165203. 753	165223. 977	165244. 202	165264. 427	165284. 652	906
907	165304. 878	165325. 103	165345. 329	165365. 555	165385. 781	165406. 008	165426. 234	165446. 461	165466. 688	165486. 916	907
908	165507. 143	165527. 371	165547. 599	165567. 827	165588. 055	165608. 284	165628. 513	165648. 742	165668. 971	165689. 201	908
909	165709. 431	165729. 661	165749. 891	165770. 121	165790. 352	165810. 583	165830. 814	165851. 045	165871. 277	165891. 508	909
910	165911. 740	165931. 972	165952. 205	165972. 437	165992. 670	166012. 903	166033. 137	166053. 370	166073. 604	166093. 838	910

	0	1	2	3	4	5	6	7	8	9	
911	166114.072	166134.306	166154.541	166174.776	166195.011	166215.246	166235.481	166255.717	166275.953	166296.189	911
912	166316.425	166336.662	166356.899	166377.136	166397.373	166417.610	166437.848	166458.086	166478.324	166498.562	912
913	166518.801	166539.040	166559.279	166579.518	166599.757	166619.997	166640.237	166660.477	166680.717	166700.958	913
914	166721.198	166741.439	166761.680	166781.922	166802.163	166822.405	166842.647	166862.889	166883.132	166903.375	914
915	166923.617	166943.861	166964.104	166984.348	167004.591	167024.835	167045.080	167065.324	167085.569	167105.813	915
916	167126.059	167146.304	167166.549	167186.795	167207.041	167227.287	167247.534	167267.780	167288.027	167308.274	916
917	167328.522	167348.769	167369.017	167389.265	167409.513	167429.761	167450.010	167470.259	167490.508	167510.757	917
918	167531.006	167551.256	167571.506	167591.756	167612.006	167632.257	167652.508	167672.759	167693.010	167713.261	918
919	167733.513	167753.765	167774.017	167794.269	167814.522	167834.774	167855.027	167875.280	167895.534	167915.787	919
920	167936.041	167956.295	167976.549	167996.804	168017.059	168037.313	168057.569	168077.824	168098.079	168118.335	920
921	168138.591	168158.847	168179.104	168199.360	168219.617	168239.874	168260.132	168280.389	168300.647	168320.905	921
922	168341.163	168361.421	168381.680	168401.939	168422.198	168442.457	168462.716	168482.976	168503.236	168523.496	922
923	168543.756	168564.017	168584.278	168604.539	168624.800	168645.061	168665.323	168685.585	168705.847	168726.109	923
924	168746.371	168766.634	168786.897	168807.160	168827.424	168847.687	168867.951	168888.215	168908.479	168928.744	924
925	168949.008	168969.273	168989.538	169009.803	169030.069	169050.335	169070.601	169090.867	169111.133	169131.400	925
926	169151.667	169171.934	169192.201	169212.468	169232.736	169253.004	169273.272	169293.540	169313.809	169334.078	926
927	169354.347	169374.616	169394.885	169415.155	169435.425	169455.695	169475.965	169496.235	169516.506	169536.777	927
928	169557.048	169577.320	169597.591	169617.863	169638.135	169658.407	169678.679	169698.952	169719.225	169739.498	928
929	169759.771	169780.045	169800.318	169820.592	169840.867	169861.141	169881.415	169901.690	169921.965	169942.240	929
930	169962.516	169982.792	170003.067	170023.343	170043.620	170063.896	170084.173	170104.450	170124.727	170145.004	930
931	170165.282	170185.560	170205.838	170226.116	170246.395	170266.673	170286.952	170307.231	170327.510	170347.790	931
932	170368.070	170388.350	170408.630	170428.910	170449.191	170469.472	170489.753	170510.034	170530.315	170550.597	932
933	170570.879	170591.161	170611.443	170631.726	170652.008	170672.291	170692.575	170712.858	170733.141	170753.425	933
934	170773.709	170793.994	170814.278	170834.563	170854.847	170875.133	170895.418	170915.703	170935.989	170956.275	934
935	170976.561	170996.848	171017.134	171037.421	171057.708	171077.995	171098.283	171118.570	171138.858	171159.146	935
936	171179.435	171199.723	171220.012	171240.301	171260.590	171280.879	171301.169	171321.459	171341.749	171362.039	936
937	171382.329	171402.620	171422.911	171443.202	171463.493	171483.785	171504.076	171524.368	171544.660	171564.953	937
938	171585.245	171605.538	171625.831	171646.124	171666.418	171686.711	171707.005	171727.299	171747.593	171767.888	938
939	171788.183	171808.478	171828.773	171849.068	171869.364	171889.659	171909.955	171930.251	171950.548	171970.844	939
940	171991.141	172011.438	172031.736	172052.033	172072.331	172092.629	172112.927	172133.225	172153.524	172173.822	940
941	172194.121	172214.420	172234.720	172255.019	172275.319	172295.619	172315.919	172336.220	172356.520	172376.821	941
942	172397.122	172417.424	172437.725	172458.027	172478.329	172498.631	172518.933	172539.236	172559.539	172579.842	942
943	172600.145	172620.448	172640.752	172661.056	172681.360	172701.664	172721.968	172742.273	172762.578	172782.883	943
944	172803.188	172823.494	172843.800	172864.106	172884.412	172904.718	172925.025	172945.332	172965.639	172985.946	944
945	173006.253	173026.561	173046.869	173067.177	173087.485	173107.794	173128.102	173148.411	173168.720	173189.030	945
946	173209.339	173229.649	173249.959	173270.269	173290.580	173310.890	173331.201	173351.512	173371.823	173392.135	946
947	173412.446	173432.758	173453.070	173473.383	173493.695	173514.008	173534.321	173554.634	173574.947	173595.261	947
948	173615.575	173635.889	173656.203	173676.517	173696.832	173717.147	173737.462	173757.777	173778.092	173798.408	948
949	173818.724	173839.040	173859.356	173879.673	173899.990	173920.307	173940.624	173960.941	173981.259	174001.576	949
950	174021.894	174042.213	174062.531	174082.850	174103.168	174123.487	174143.807	174164.126	174184.446	174204.766	950
951	174225.086	174245.406	174265.727	174286.047	174306.368	174326.689	174347.011	174367.332	174387.654	174407.976	951
952	174428.298	174448.621	174468.943	174489.266	174509.589	174529.912	174550.236	174570.560	174590.883	174611.208	952
953	174631.532	174651.856	174672.181	174692.506	174712.831	174733.156	174753.482	174773.808	174794.134	174814.460	953
954	174834.786	174855.113	174875.440	174895.767	174916.094	174936.421	174956.749	174977.077	174997.405	175017.733	954
955	175038.062	175058.390	175078.719	175099.048	175119.378	175139.707	175160.037	175180.367	175200.697	175221.028	955
956	175241.358	175261.689	175282.020	175302.351	175322.683	175343.014	175363.346	175383.678	175404.010	175424.343	956
957	175444.675	175465.008	175485.341	175505.675	175526.008	175546.342	175566.676	175587.010	175607.344	175627.679	957
958	175648.014	175668.349	175688.684	175709.019	175729.355	175749.691	175770.027	175790.363	175810.699	175831.036	958
959	175851.373	175871.710	175892.047	175912.385	175932.722	175953.060	175973.398	175993.737	176014.075	176034.414	959
960	176054.753	176075.092	176095.431	176115.771	176136.111	176156.451	176176.791	176197.131	176217.472	176237.812	960
961	176258.154	176278.495	176298.836	176319.178	176339.520	176359.862	176380.204	176400.546	176420.889	176441.232	961
962	176461.575	176481.918	176502.262	176522.606	176542.950	176563.294	176583.638	176603.983	176624.327	176644.672	962
963	176665.017	176685.363	176705.708	176726.054	176746.400	176766.746	176787.093	176807.440	176827.786	176848.133	963
964	176868.481	176888.828	176909.176	176929.524	176949.872	176970.220	176990.568	177010.917	177031.266	177051.615	964
965	177071.965	177092.314	177112.664	177133.014	177153.364	177173.714	177194.065	177214.416	177234.767	177255.118	965
966	177275.469	177295.821	177316.173	177336.525	177356.877	177377.229	177397.582	177417.935	177438.288	177458.641	966
967	177478.994	177499.348	177519.702	177540.056	177560.410	177580.765	177601.120	177621.475	177641.830	177662.185	967
968	177682.540	177702.896	177723.252	177743.608	177763.965	177784.321	177804.678	177825.035	177845.392	177865.750	968
969	177886.107	177906.465	177926.823	177947.181	177967.540	177987.898	178008.257	178028.616	178048.975	178069.335	969
970	178089.694	178110.054	178130.414	178150.775	178171.135	178191.496	178211.857	178232.218	178252.579	178272.941	970
971	178293.302	178313.664	178334.026	178354.389	178374.751	178395.114	178415.477	178435.840	178456.204	178476.567	971
972	178496.931	178517.295	178537.659	178558.023	178578.388	178598.753	178619.118	178639.483	178659.849	178680.214	972
973	178700.580	178720.946	178741.312	178761.679	178782.045	178802.412	178822.779	178843.147	178863.514	178883.882	973
974	178904.250	178924.618	178944.986	178965.355	178985.723	179006.092	179026.461	179046.831	179067.200	179087.570	974
975	179107.940	179128.310	179148.680	179169.051	179189.422	179209.793	179230.164	179250.535	179270.907	179291.279	975
976	179311.651	179332.023	179352.395	179372.768	179393.141	179413.514	179433.887	179454.260	179474.634	179495.008	976
977	179515.382	179535.756	179556.130	179576.505	179596.880	179617.255	179637.630	179658.006	179678.381	179698.757	977
978	179719.133	179739.510	179759.886	179780.263	179800.640	179821.017	179841.394	179861.772	179882.149	179902.527	978
979	179922.906	179943.284	179963.662	179984.041	180004.420	180024.799	180045.179	180065.558	180085.938	180106.318	979
980	180126.698	180147.078	180167.459	180187.840	180208.221	180228.602	180248.983	180269.365	180289.747	180310.129	980

Table of 2n ln n *for values of* n *from 1 to 10,000*—Continued

	0	1	2	3	4	5	6	7	8	9	
981	180330. 511	180350. 893	180371. 276	180391. 659	180412. 042	180432. 425	180452. 809	180473. 192	180493. 576	180513. 960	981
982	180534. 344	180554. 729	180575. 114	180595. 498	180615. 883	180636. 269	180656. 654	180677. 040	180697. 426	180717. 812	982
983	180738. 198	180758. 585	180778. 971	180799. 358	180819. 745	180840. 133	180860. 520	180880. 908	180901. 296	180921. 684	983
984	180942. 072	180962. 461	180982. 849	181003. 238	181023. 627	181044. 017	181064. 406	181084. 796	181105. 186	181125. 576	984
985	181145. 967	181166. 357	181186. 748	181207. 139	181227. 530	181247. 921	181268. 313	181288. 705	181309. 097	181329. 489	985
986	181349. 881	181370. 274	181390. 667	181411. 060	181431. 453	181451. 846	181472. 240	181492. 634	181513. 028	181533. 422	986
987	181553. 816	181574. 211	181594. 606	181615. 001	181635. 396	181655. 791	181676. 187	181696. 583	181716. 979	181737. 375	987
988	181757. 771	181778. 168	181798. 565	181818. 962	181839. 359	181859. 757	181880. 154	181900. 552	181920. 950	181941. 348	988
989	181961. 747	181982. 146	182002. 544	182022. 943	182043. 343	182063. 742	182084. 142	182104. 542	182124. 942	182145. 342	989
990	182165. 743	182186. 143	182206. 544	182226. 945	182247. 347	182267. 748	182288. 150	182308. 552	182328. 954	182349. 356	990
991	182369. 758	182390. 161	182410. 564	182430. 967	182451. 371	182471. 774	182492. 178	182512. 582	182532. 986	182553. 390	991
992	182573. 795	182594. 199	182614. 604	182635. 009	182655. 415	182675. 820	182696. 226	182716. 632	182737. 038	182757. 444	992
993	182777. 851	182798. 258	182818. 664	182839. 072	182859. 479	182879. 886	182900. 294	182920. 702	182941. 110	182961. 519	993
994	182981. 927	183002. 336	183022. 745	183043. 154	183063. 563	183083. 973	183104. 383	183124. 793	183145. 203	183165. 613	994
995	183186. 024	183206. 434	183226. 845	183247. 257	183267. 668	183288. 079	183308. 491	183328. 903	183349. 315	183369. 728	995
996	183390. 140	183410. 553	183430. 966	183451. 379	183471. 793	183492. 206	183512. 620	183533. 034	183553. 448	183573. 862	996
997	183594. 277	183614. 692	183635. 107	183655. 522	183675. 937	183696. 353	183716. 769	183737. 185	183757. 601	183778. 017	997
998	183798. 434	183818. 851	183839. 267	183859. 685	183880. 102	183900. 520	183920. 937	183941. 355	183961. 774	183982. 192	998
999	184002. 611	184023. 029	184043. 448	184063. 867	184084. 287	184104. 706	184125. 126	184145. 546	184165. 966	184186. 387	999

2 (10000 ln 10000) = 184206.807

(Paper 66B4–87)

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Publications of the National Bureau of Standards*

Selected Abstracts

Strong blast waves in spherical, cylindrical and plane shocks, D. L. Jones, *Phys. of Fluids* **4**, *No. 9 (Sept. 1961)*.
The integrated energy parameter B for blast wave analysis is calculated for monatomic and diatomic gases. Three geometries; spherical, cylindrical and plane; are considered. Where possible, comparison is made with previously published values of B.

Approximations to the moments of the sample median, M. M. Siddiqui, *Ann. Math. Stat.* **33**, *No. 1, 157–168 (Mar. 1962)*.
In this paper a numerical study of Chu and Hotelling's method of approximating to the moments of the sample median will be made. With an introductory outline of their method in Section 2, we will proceed to apply it to various distributions, and will evaluate the degree of accuracy that can be conveniently obtained by means of it in each particular case. The numerical results will be presented in tabular form.

Kinetic equation for plasmas with collective and collisional correlations, C. M. Tchen, *Proc. Fifth Conf. on Ionization Phenomena in Gases, Munich, pp. 825–841 (North Holland Publ. Co., Amsterdam, The Netherlands, 1961)*.
A kinetic equation for plasmas is derived from the so-called BBGKY equations. The effects of the double and triple correlations are investigated. Emphasis is given to the formulation of the long range collective force, the collision force of shorter range, the mixing force between the two ranges, and the shielding mechanism. A consistent study is advanced about the time development of the double correlation function. In view of applications to plasma oscillations, the distribution functions are perturbed. The unperturbed double correlation and its perturbation are found, the latter as the solution of a singular integral equation.

Propagation of solar particles and the interplanetary magnetic field, C. S. Warwick, *J. Geophys. Research* **67**, *No. 4, 1333–1346 (Apr. 1962)*.
Effects on the propagation of cosmic-ray particles and of solar protons associated with PCA (polar cap absorption) indicate the presence of an interplanetary magnetic field. The effect of this field is greatest for particles of energy of about 10^9 eV, indicating that particles of lower energy, associated with PCA, propagate not as individual particles, but as a group with kinetic energy density comparable to the magnetic energy density of regions of the interplanetary field. A model of this field is proposed to explain various characteristics of the solar particles. This model is found to account for the main features of solar modulation of galactic cosmic rays.

Modular forms whose coefficients possess multiplicative properties (II), M. Newman, *Ann. of Math.* **75**, *242–250 (Mar. 1962)*.
By means of the elliptic modular functions many number theoretic identities are obtained. For example, if $r_s(n)$ denotes the number of representations of n as the sum of s squares, and p is an odd prime, then it is shown that

$$r_s(np^2)=\alpha r_s(n)-p^{s-2}rs\left(\frac{n}{p^2}\right),$$

where

$$\alpha=1+p^{s-2}-(-1)^{\frac{(p-1)(s-1)}{4}}\,p^{\frac{s-3}{2}}\left(\frac{n}{p}\right),$$

$\left(\frac{n}{p}\right)$ is the Legendre-Jacobi symbol, and $s=1, 3, 5, 7$.

A calculus for factorial arrangements, B. Kurkjian and M. Zelen, *Ann. Math. Stat.* **33**, *600–619 (June 1962)*.
This paper introduces a special calculus for factorial experiment designs. The calculus applies to the general case of asymmetric factorial arrangements and is not restricted to symmetric factorial designs as is the current theory which relies on the theory of finite projective geometry. The concise notation and operations of this calculus point up the relationship of treatment combinations to interactions and the effect of patterns of arrangements on the distribution of relevant quantities. One aim of the calculus is to carry out complex manipulations and operations with relative ease. The calculus enables many large order arithmetic operations, necessary for analyzing factorial designs, to be partly carried out by logical operations. This should be of importance in programming the analysis of factorial designs on high speed computers.
Some of this work is related to the work of Bose and Bose and Connor on the analysis of fractionally replicated designs of the mixed 2^m3^n series. In their work they have also constructed a kind of calculus for simplifying the solution of treatment estimates.
The principal new results of this paper, aside from the new notation, is the development of the polynomial regression model. In particular, the use of the calculus enables one to write the inverse matrix of the normal equations as a partitioned matrix which only requires inverting matrices of smaller order.

A note on normal matrices, M. Marcus and N. Khan, *Can. Math. Bull.* **4**, *23–27 (1961)*.
It is proved that only normal matrices A have the following property: The eigenvalues of a linear function of A and A* are orthogonally related to the same linear function in the eigenvalues of A.

The evolution of concepts and languages of computing, R. D. Elbourn and W. H. Ware, *Proc. IRE* **50**, *No. 5, 1059–1066 (May 1962)*.
Digital computers are opening exciting new possibilities for progress in language translation, information retrieval, psychological modeling, problem solving and theorem proving. These have resulted not because of their microsecond arithmetic speed but because of their ability to manipulate symbols: to read, write, store, compare, and replace symbols and to follow different courses of action according to differences between symbols. Thus, language in a general sense is a common aspect of these applications.
Programmers have been extending the usefulness of computers through the evolutionary development of most artfully conceived languages. Recently, mathematicians and logicians have been proving theorems about formal languages, while linguists have been discovering laws that humans instinctively observe whenever they use natural language. Fruitful cross-pollination among these endeavors now promises greatly accelerated progress in determining whether symbol manipulation is for information processing applications what numerical analysis is for arithmetic applications.
This paper first reviews the evolution of programming languages from the early days when all programming was done in machine languages, through symbolic coding systems, interpreters, assemblers, generators, and compilers, to the recently developed list processing languages. Then the applications of these languages to game playing, problem solving, theorem proving, and behavior and biological modeling have been described briefly. Finally, in anticipation of extending the capability of computers to accept, use and generate

natural languages, the paper concludes with an introduction to some of the contemporary work on formal language theory, including a discussion of six families of abstract languages and their practical implementation.

Displacement and strain-energy distribution in a longitudinally vibrating cylindrical rod with a viscoelastic coating, P. Hertelendy, *J. Appl. Mech.* **29,** *Series E, 47–52 (Mar. 1962).*
A numerical solution by R. M. Davies of the Pochhammer frequency equation is used to determine the displacement and strain energy distribution across the cross-section of an infinite elastic circular cylindrical rod for a number of wave lengths of the first, second, and third modes of symmetrical longitudinal wave propagation. With these results the effect of a thin uniform layer of viscoelastic material is investigated. The four viscoelastic parameters of the coating are reduced to one in the definition and computation of upper and lower bounds of the loss factor, and the application of results to experimental work is discussed.

Tchebycheff approximation by functions unisolvent of variable degree, J. R. Rice, *Trans. Am. Math. Soc.* **99,** *298–302 (1961).*
The definition of unisolvence (T. S. Motzkin, Bull. Amer. Math. Soc. **22** (1949) 789–793) may be generalized so as to include a much larger class of functions and to preserve the results of unisolvent functions. Let F be a function of N+1 real variables, $x\Sigma[0,1]$ and $(a_1,a_2, \ldots, a_n)$ Σp where p is a subset of E_n. Let a denote the point $(a_1,a_2, \ldots, a_n)$. Further let F be a continuous function of all of its variables and let $F(a,x)=F(a',x)$ for all $x\Sigma[0,1]$ imply that $a=a'$. F is said to be unisolvent of degree N at a point $a^*\Sigma p$ if i) there is no $a\Sigma p$ such that $F(a,x_i)=F(a^*,x_i)$ for n distinct x_i, ii) given n distinct x_i and $\Sigma>0$ then there exists $\delta(a^*_1,x, \ldots, x_n,\Sigma)>0$ such that for any set $\{y_i|F(a^*,x_i)-y_i|<\delta\}$ there is $a'\Sigma p$ for which $F(a',x_i)=y_i$ and $\max_{x\Sigma[0,1]}|F(a',x)-F(a^*,x)|<\Sigma$. This class of functions includes rational functions, linear combinations of exponentials with variable exponent and many other elementary functions. Existence and uniqueness are discussed and a theorem on the characterization of best Tchebycheff approximations is established. *Theorem:* Let F be unisolvent of degree n at a* and let f(x) be a function continuous on [0,1]. Then a necessary and sufficient condition that $F(a^*,x)$ be a best approximation to f(x) is that $\max_{x\Sigma[0,1]}|F(a^*,x)-f(x)|$ be assumed at least N+1 times with alternating sign.

Best approximations and interpolating functions, J. R. Rice, *Trans. Am. Math. Soc.* **101,** *477–498 (Dec. 1961).*
A function $F(a,x)$ depending on n parameters is said to be an interpolating function of $f(x)$ if $F(a,x)$ interpolates $f(x)$ in at least n points. Interpolating functions are often used as approximations to $f(x)$. Motzkin and Walsh have recently investigated the extent of the relationship between best polynomial approximations in an L_p norm and interpolating polynomials. In this paper this relationship is investigated more deeply and in much more general situations. The general problem is to say when a best approximation is also an interpolating function and when an interpolating function is a best approximation in a given norm. Results are obtained for varisolvent approximating functions as well as approximating functions of the form $\Sigma_{i=1}^{n} a_i\phi_i(x)$. The norms considered are more general then the weighted L_p norms.

On the theory of diffraction grating interferometers, H. Mendlowitz and J. A. Simpson, *J. Opt. Soc. Am.* **52,** *No. 5, 520–524 (May 1962).*
One-dimensional diffraction grating theory is developed in vector notation. First order expressions are derived for the effects of the rotation of the grating about an arbitrary axis upon the diffracted beam. These results are applied to the three-grating-interferometer whose characteristics are given in some detail. Generalization to moire patterns or to any number of gratings with arbitrary separations is indicated.

Traces of products of angular momentum matrices, I. Cartesian basis, E. Ambler, J. C. Eisenstein, and J. F. Schooley, *J. Math. Phys.* **3,** *No. 1, 118–130 (Jan.–Feb. 1962).*
Closed formulas are given for evaluating Tr $J_a^pJ_b^qJ_c^r \ldots$ where a,b,c . . . are equal to x, y or z and p, q, r . . . are non-negative integers for which $p+q+r+ \ldots \leq 10$. All possible combinations of the angular momentum components for $p+q+r+ \ldots \leq q$ are included. Numerical values of the traces are given for $J=\frac{1}{2}, 1, \ldots 10$. The procedures used in evaluating the traces are described.

Digital pattern recognition by moments, F. L. Alt, *J. Assoc. Computing Mach.* **9,** *No. 2, 240–258 (Apr. 1962).*
To identify plain black-and-white patterns such as alphabetic characters, by means of a digital computer, the first few moments of each pattern are computed and compared to those of prototype patterns. Certain combinations of the moments are used which are invariant under frequently used transformations of patterns. Experiments indicate that such a process, using a modest number of sample points on each pattern, and computing moments only up to the sixth order, is adequate to differentiate, e.g., between any two characters of the alphabet. The general problem of classifying items characterized by a finite set of numbers (in this case, patterns characterized by the set of moments up to order 6) is discussed and a tentative solution proposed.

Other NBS Publications

Journal of Research 66A (Phys. and Chem.) No. 5 (Sept.–Oct. 1962) 70 cents.

Calorimetric calibration of an ionization chamber for determination of X-ray total beam energy. J. S. Pruitt and S. R. Domen.
Zinc oxide as a standard substance in the solution calorimetry of portland cement. E. S. Newman.
Nuclear optical model analysis of neutron elastic scattering for calcium. R. S. Caswell.
Pyrolysis of some polyvinyl polymers at temperatures up to 1,200 °C. S. Straus and S. L. Madorsky.
Lattice frequencies and rotational barriers for inorganic carbonates and nitrates from low temperature infrared spectroscopy. R. A. Schroeder, C. E. Weir, and E. R. Lippincott.
Foreign gas broadening of the lines of hydrogen chloride and carbon monoxide. E. K. Plyler and R. J. Thibault.
Monolayers of adipate polyesters at air-liquid interfaces. W. M. Lee, R. R. Stromberg, and J. L. Shereshefsky.

Journal of Research 66C (Eng. and Instr.) No. 4 (Oct.–Dec. 1962) 75 cents.

An ultra-high speed image dissecting camera for photographing strong shock waves. K. B. Earnshaw and C. M. Benedict.
Biprism method of determining the equivalent focal length of flat field lenses. W. R. Darling.
Effect of air drag on the motion of a filament struck transversely by a high-speed projectile. F. L. McCrackin.
A precision noise spectral density comparator. C. M. Allred.
Stresses in a plate uniformly compressed over portions of its two opposite edges. M. Chi and W. D. Kroll.
Studies of the stress-corrosion cracking of low-carbon steels. H. L. Logan.
A dual centrifuge for generating low-frequency sinusoidal accelerations. R. O. Smith, E. A. Willis, and J. S. Hilten.
Rotational micromanometers. K. Lofquist.
Study of gypsum plasters exposed to fire. J. V. Ryan.

Journal of Research 66D (Radio Prop.) No. 5 (Sept.–Oct. 1962) 70 cents.

Theory of magneto-telluric fields. J. R. Wait.
Propagation characteristics of magneto-ionic plasma columns. D. Formato and A. Gilardini.
Dielectric loading of electric dipole antennas. J. Galejs.
Possible influence of the ionosphere on the impedance of a ground-based antenna. J. R. Wait.
Some statistical theory for the analysis of radio propagation data. M. M. Siddiqui.
Auroral sporadic-E ionization. R. D. Hunsucker and L. Owren.

Comparative study of the correlation of seasonal and diurnal cycles of transhorizon radio transmission loss and surface refractivity. B. R. Bean.
Enhancement of the lunar tide in the noon critical frequency of the F_2 layer over the magnetic equator. R. G. Rastogi.
Scattering from a conducting sphere embedded in a semi-infinite dissipative medium. J. Galejs.
High-frequency scattering from a coated sphere. V. H. Weston and R. Hemenger.
Propagation of spherical waves through an ionosphere containing anisotropic irregularities. K. C. Yeh.

Journal of Reseach 66D (Radio Prop.) No. 6 (Nov.–Dec. 1962) 70 cents.
RF impedance probe measurements of ionospheric electron densities. J. A. Kane, J. E. Jackson, and H. A. Whale.
Methods for applying numerical maps of ionospheric characteristics. W. B. Jones and R. M. Gallet.
Very-low-frequency radio propagation in the ionosphere. D. W. Swift.
Prolonged space-wave fadeouts in tropospheric propagation. A. P. Barsis and M. E. Johnson.
Range-error compensation for a troposphere with exponentially varying refractivity. J. J. Freeman.
On the geometrical optics of curved surfaces with periodic impedance properties. C. J. Marcinkowski and L. B. Felsen.
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Conversion of the amplitude-probability distribution function for atmospheric radio noise from one bandwidth to another. A. D. Spaulding, C. J. Roubique, and W. Q. Crichlow.
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Structure shielding against fallout radiation from nuclear weapons, L. V. Spencer, NBS Mono. 42 (June 1, 1962) 75 cents.
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Determination of total X-ray beam energy with a calibrated ionization chamber, J. S. Pruitt and S. R. Domen, NBS Mono. 48 (June 5, 1962) 20 cents.
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